Derating of Induction Motors Due to Waveform Distortion

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***Abstract*—Electrical motors are designed on the basis of balanced three-phase sinusoidal input voltage. Nonsinusoidal voltage has detrimental effects on induction motor performance, and derating of the machine is required. IEEE Standard 519 suggests that no derating of the motor would be necessary for a harmonic content of up to 5%. The derating of induction motors due to harmonic distortion is discussed in detail.**

Introduction

THE OUTPUT of an induction motor depends mainly on heating, and the motor's life is shortened by overheating. The temperature rise resulting from losses is, therefore, a major factor in determining the machine output rating. The presence of harmonics in the applied voltage can cause excessive heating.

The amount of voltage distortion, measured by a "distortion factor" (DF) and defined by IEEE Standard 519 [1] as

$$\mathrm{DF} = \left[\frac{\text{sum of squares of amplitudes of all harmonic voltages}}{\text{square of amplitude of fundamental voltage}}\right]^{1/2}, \quad (1)$$

is used to establish harmonic limits. On industrial power systems, the voltage distortion is limited to 5%. However, no limit is specified in regard to the individual harmonic content. Derating of "NEMA design B" induction motors of different output ratings and for two types of enclosures (drip-proof and totally enclosed) due to different cases of harmonic distortions are discussed in this paper.

Analytical Background

General Assumptions

1) The motors are nonskewed, Y-connected, and ungrounded.

2) The analysis is limited to full-load steady-state operating conditions.

3) The principle of superposition applies.

Equivalent Circuit Parameters

The simplified equivalent circuit of a three-phase induction motor is shown in Fig. 1. It is to be remembered that this equivalent circuit does not take into account any time or space harmonics. The various resistances and reactances are referred to the stator winding and are expressed in per unit (pu) at the machine base. These parameters (R_s, R_R, $X_m = wL_m$, X_s, X_R, R_c) are assumed to be constants. This is true only for a given operating condition. They vary with changes in motor current, speed, voltage, and temperature [2].

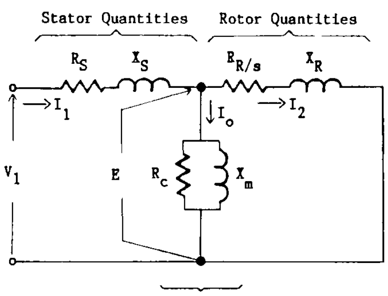

Fig. 1. Equivalent circuit of induction motor.

Motor Losses

The total motor losses (P_{diss}) consist of iron, winding, mechanical, and stray losses.

Iron Losses (P_{fe}): For an alternating magnetic field, losses that occur in the iron consist of hysteresis loss and eddy current loss. In general, for sinusoidal flux, the losses in Watt/kg of iron (for a given lamination thickness) can be expressed as

$$P_{\text{fe}} = k_h f B_m^2 + k_e (f B_m)^2 \quad (2)$$

where the first term accounts for the hysteresis loss and the second term for the eddy current loss. The constants k_h and k_e depend on the properties of the material. B_m is the maximum flux density and is proportional to the air-gap voltage E. If the flux density B_m is uniform over the cross-sectional area A of the core,

$$B_m = \frac{\phi_m}{A} = \frac{E}{4.44 f N A} = \frac{C_m E}{f} \quad (3)$$

where E is the rms value of the air-gap voltage, C_m is a machine constant, and f is the frequency. Substituting (3) in (2) gives

$$P_{\text{fe}} = \left[k_h (1/f) + k_e\right] (E C_m)^2. \quad (4)$$

Winding Losses (P_{cu1} and P_{cu2}): These losses are stator

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and rotor I^2R losses caused by the current flowing through the respective winding.

Mechanical Losses (P_{mech}): The mechanical losses comprise of friction and windage losses. They are approximately proportional to the square of the speed and to the contact surface area. These losses will be assumed to be unaffected by voltage harmonic distortion.

Stray-Load Losses (P_{stray}): These are additional iron and eddy current losses caused by the increase in air-gap leakage flux with load, and by high-frequency pulsation fluxes. These losses can be divided into six components as follows:

1) the eddy current loss in the stator copper W_c due to slot leakage flux (normally neglected);
2) the losses in the motor end structure W_e due to end leakage flux;
3) the high-frequency rotor and stator surface losses W_s due to zig-zag leakage flux;
4) the high-frequency tooth pulsation and rotor I^2R losses W_z, also due to the zig-zag leakage flux;
5) the six-times-frequency (for three-phase machines) rotor I^2R losses W_b due to circulating currents induced by the stator belt leakage flux;
6) The extra iron losses in motors with skewed slots W_k due to skew leakage flux (neglected here because of the nonskewed assumption).

The equations for these components given in [3] are (with some changes in notation)

$$W_e = C_e I_1^2 f_1 \tag{5}$$

$$W_s = C_s (I_1/I_0)^2 B_g^2 \tag{6}$$

$$W_z = C_z k_{rs} R_R (C_0 I_0^2 + C_1 I_1^2) \tag{7}$$

$$W_b = C_b k_{rm} R_R I_1^2 \tag{8}$$

where C_e, C_s, C_z, C_b, C_0, and C_1 are constants that depend on the machine and other empirical factors, k_{rs} and k_{rm} are the skin effect coefficients for the rotor bars at the stator slot harmonic frequency and at the phase belt frequency, respectively, B_g is the average flux density over the effective air-gap area, I_0 is the no-load current, I_1 is the stator current, and f_1 is the line frequency. For typical standard NEMA design B machines operating at full load, the losses can be distributed as [4]

$$\begin{aligned} P_{mech} &= 0.09 \\ P_{fe} &= 0.20 \\ P_{cu1} &= 0.37 \\ P_{cu2} &= 0.18 \\ P_{stray} &= 0.16 \\ \hline P_{diss} &= 1.00. \end{aligned}$$

Thermal Stress [2], [5]

The allowable hot-spot temperature in the stator winding determined by the insulation class governs the output of a machine. Assuming a lumped parameter approach, Figs. 2 and 3 show the simplified thermal equivalent circuit of a drip-proof radially cooled and a totally enclosed fan-cooled machine.

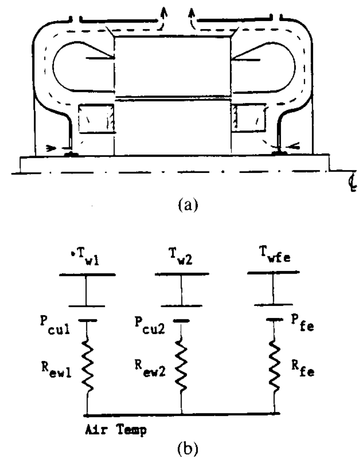

Fig. 2. Drip-proof radially cooled squirrel-cage motor. (a) Air flow. (b) Thermal network.

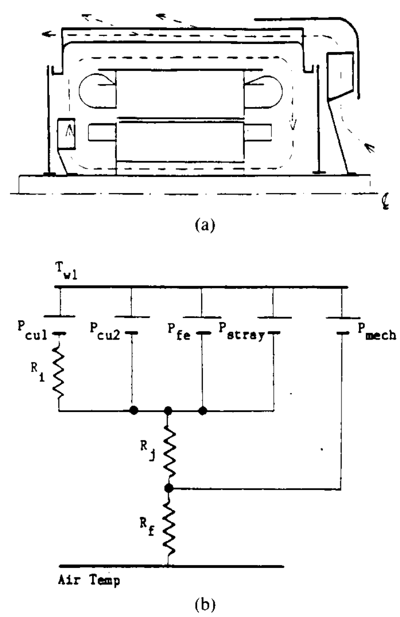

Fig. 3. Totally enclosed fan-cooled squirrel-cage motor. (a) Air flow. (b) Thermal network.

The temperature rise of the stator winding T_{w1} is then (for a drip-proof radially cooled machine)

$$T_{w1} = P_{cu1} R_{ew1} \tag{9}$$

where R_{ew1} is the thermal resistance of the stator end-winding. For a totally enclosed fan-cooled machine, the temperature rise is

$$\begin{aligned} T_{w1} = P_{cu1} R_i &+ (P_{cu1} + P_{cu2} + P_{fe} + P_{stray}) R_j \\ &+ (P_{cu1} + P_{cu2} + P_{fe} + P_{stray} + P_{mech}) R_f \end{aligned} \tag{10}$$

where R_i, R_j, and R_f are the thermal resistances of the slot insulation, stator backiron (including air-gap between the stator core and outside frame of the motor), and outside frame and moving air, respectively.

Derating of Induction Motors Due to Harmonics

Voltage Waveshape

The nonsinusoidal supply voltage can be expressed in a general form as

$$v(t) = \sqrt{2}\left[V_1 \sin wt + \sum_{k=2}^{n} V_k \sin(kwt + \theta_k)\right] \quad (11)$$

where V_1 is the fundamental voltage, V_k represents harmonic voltages of order k, and θ_k is the harmonic phase angle. The fundamental and the $4, 7, 10, 13, \cdots, [3n + 1]$, $n = 0, 1, 2, \cdots$, order voltage harmonics contribute to a rotating magnetomotive force (MMF) in the direction of motion and hence results in the development of a positive torque. The $2, 5, 8, 11, \cdots, [3n + 2]$, $n = 0, 1, 2, \cdots$, order of harmonics results in a rotating MMF in a direction opposite to the direction of motion of the rotor and hence contributes to a negative torque. The $3, 6, 9, 12, \cdots, [3n + 3]$, $n = 0, 1, 2, \cdots$ order of harmonics produces no rotating MMF and, therefore, no torque.

Equivalent Circuit Parameters

Based on assumption 3), the temperature rise of the motor can be computed by superposition as if a series of independent generators were supplying the motor. Each generator would represent one of the voltage terms in (11). Each of these voltages would produced stator and rotor currents.

Evaluation of the reactances and resistances in these series of equivalent circuits ought to be done at the corresponding harmonic frequencies.

The actual frequency of the current in the stator is $[k \cdot f_1]$ and in the rotor $[k \cdot f_1 \cdot s_k]$, where f_1 is the fundamental frequency and s_k is the slip for the kth harmonic. At these frequencies, skin and proximity effects should be considered in inductances and resistances. The synchronous speed corresponding to the applied frequency $[k \cdot f_1]$ is $[k \cdot N_s]$, N_s being the synchronous speed in revolutions per minute corresponding to the fundamental (i.e., $N_s = 120 \cdot f_1 / p$, p is the number of poles). Therefore, the slip s_k at any speed N_r of the rotor is given by

$$s_k = \frac{kN_s + N_r}{kN_s}. \quad (12)$$

The plus sign has been used to account for the fact that some harmonics result in rotating MMF's in the same direction as the motion of the rotor, while others result in rotating MMF's in the opposite direction of the motion of the rotor. In terms of the slip s (corresponding to the fundamental),

$$N_r = (1 - s) N_s. \quad (13)$$

Therefore,

$$s_k = \frac{kN_s + (1 - s)N_s}{kN_s} = \frac{k + (1 - s)}{k}. \quad (14)$$

To arrive at appropriate values for the inductances in the circuit with the motor operating with nonsinusoidal voltage, the effects of harmonic voltage and currents on the degree of saturation must also be considered. The effect of saturation is to limit the flux in the iron of the flux paths, causing a reduction in the inductances.

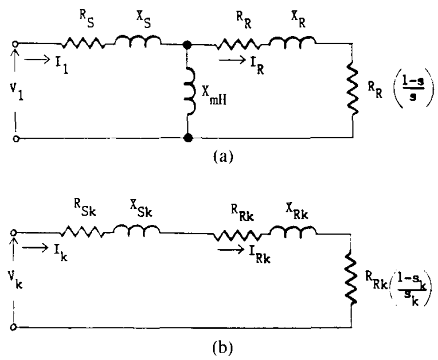

Fig. 4. Equivalent circuit for induction motor with nonsinusoidal excitation. (a) Fundamental. (b) k_h harmonic.

Effect of Saturation

To determine the effect of saturation in the no-load current due to harmonic distortion, a new modified magnetizing reactance is calculated as

$$X_{mH} = X_m / M_f \quad (15)$$

where the factor M_f is defined as

$$M_f = \sqrt{\frac{1}{T}\sum_{n=0}^{n=T} i_{ma}^2(\Delta t)} \Bigg/ \sqrt{\frac{1}{T}\sum_{n=0}^{n=T} i_m^2(\Delta t)} \quad (16)$$

where i_m and i_{ma} are the instantaneous magnetizing current corresponding to the flux density wave for normal and abnormal conditions, respectively.

Results of the increase in no-load current due to saturation were verified experimentally by lab tests on a 2-hp induction motor [6].

Determination of Harmonic Currents

When the motor is running near the fundamental synchronous speed, the harmonic equivalent circuit is quite similar to the locked rotor equivalent circuit for the particular harmonic being considered. The magnetizing branch may be neglected since the magnetizing reactance for the kth harmonic (kX_m) is much greater than the rotor leakage impedance for the kth harmonic (Z_{Rk}). For a similar reason, the resistors R_c and R_{ck} representing core (and mechanical) losses for the fundamental and the different harmonics are also neglected. Fig. 4(a) shows the simplified equivalent circuit for the fundamental component and Fig. 4(b) for the kth harmonic. The kth harmonic current is then given as

$$I_k = \frac{V_k}{\left[(R_{Sk} + R_{Rk}/s_k)^2 + (X_{Sk} + X_{Rk})^2\right]^{1/2}} \quad (17)$$

where V_k is the voltage due to the kth harmonic, R_{Sk} and R_{Rk} the stator and rotor resistances and X_{Sk} and X_{Rk} the stator and rotor reactances for the kth harmonic. The total harmonic current is

$$I_h = \sqrt{\sum_{k=2}^{n} I_k^2}. \tag{18}$$

Motor Losses

Iron Losses: The presence of time harmonics results in higher saturation of the magnetic paths. Consequently, iron losses will increase. This increase can be estimated by substituting the corresponding harmonic voltages and frequencies in (4).

Winding Losses: Assuming the skin effect in the stator winding to be negligible, the stator copper loss on a nonsinusoidal supply is proportional to the square of the total rms current. If R_S is the stator resistance, the loss per phase is

$$P_{\text{cu}1} = I_{\text{rms}}^2 R_s \tag{19}$$

$$I_{\text{rms}} = \sqrt{I_1^2 + I_h^2}. \tag{20}$$

Then

$$P_{\text{cu}1} = R_s(I_1^2 + I_h^2) \tag{21}$$

where $I_1^2 R_s$ represents the loss due to the fundamental and $I_h^2 R_s$ accounts for the losses due to harmonics.

When the rotor conductor depth is appreciable (as in large machines) skin effect should be taken into account. Loss due to each harmonic must be considered separately and then added. In the case of a deep bar machine rotor, the total rotor copper loss per phase is

$$P_{\text{cu}2} = \sum_{k=2}^{n} I_{Rk}^2 R_{Rk} + I_R^2 R_R. \tag{22}$$

Here, the first term represents the loss due to harmonics, and the second term will give the rotor copper loss due to the fundamental. In order to maintain constant output torque, the last term of (22) will vary an amount given by (23), which is proportional to the resultant torque produced by the harmonic currents:

$$P_{Tk} = \sum_{k=2}^{n} I_{Rk}^2 R_{Rk}/s_k. \tag{23}$$

Stray-Load Losses: Voltage harmonics significantly affect these losses. They can be estimated by adapting (5)-(8) to the motor with harmonics as follows:

$$W_e = C_e\left[I_1^2 + \sum_{k=2}^{n} (I_k^2)k(1+s_k)\right]f_1 \tag{24}$$

$$W_{s1,2} = C_s(I_1'/I_0')^2 B_g'^2 \tag{25}$$

$$W_z = C_z k_{rs} R_R (C_0 I_0'^2 + C_1 I_1'^2) \tag{26}$$

$$W_b = C_b k_{rm} R_R I_1'^2 \tag{27}$$

where I_0' is the no-load current corrected for saturation and I_1' is the total stator current including harmonics.

Results and Discussions

Using the equations derived in the preceding sections, a computer program is developed. The temperature rise of induction motors of different ratings (Table I) and two types of enclosures are determined for three different cases of 5% voltage distortion (Table II). Triplen harmonics are neglected in the calculations.

TABLE I
Three-Phase Induction Motor Characteristics [7]

Rating (hp)	Full-Load Slip (%)	Resistances and Reactances in Per Unit Based on Full-Load kVA and Rated Voltage R_S	R_R	X_m	X_S	X_R	Rotor Bar Height (mm)
Up to 5	4.5	0.055	0.055	1.9	0.048	0.072	15
5-25	3.0	0.040	0.04	2.6	0.064	0.096	25
25-200	2.5	0.030	0.03	3.2	0.068	0.102	35
200-1000	1.75	0.025	0.02	3.6	0.068	0.102	45

TABLE II
Harmonic Content of Waveforms Used

Waveform Harmonic k	Voltage in % of Fundamental a	b	c
2	5.00		0.17
3			0.21
4			0.26
5		5.00	3.41
6			0.87
7			0.25
8			0.26
9			0.61
10			0.26
11			0.56
12			0.43
13			1.21
17			0.87
19			0.78
DF =	5%	5%	5%

The thermal resistances R_i, R_j, and R_f are determined for a 5-hp and for a 100-hp machine. The values found for the 5-hp machine are used as representatives of machines ranging from 0 to 25 hp, and those found for the 100-hp machine for machines ranging from 25 to 1000 hp. Insulation class B is assumed for all cases.

Slot leakage inductance reduction due to harmonic distortion is considered equal to 10% for all waveforms and rotor slot leakage inductance is taken to be 30% of total rotor leakage for all machines. In all the computations, an ANSI M-22 steel and 1.0-T maximum air-gap magnetic flux density is used.

The results of the increases in temperature with respect to full-load normal operating conditions defined by

$$\%(\Delta T) = \frac{T_H - T_N}{T_N} * 100 \tag{28}$$

where T_H and T_N are the hot-spot temperature of the machine when supplied with nonsinusoidal (or harmonic) and sinusoidal voltages, respectively, are shown in Figs. 5 and 6.

Figs. 5 and 6 reveal that the second harmonic (case a) in a nonsinusoidal voltage has the most pronounced effect on the temperature rise. The fifth harmonic only or the assorted higher order harmonics of small magnitudes (cases b and c) do not have any appreciable effect on the temperature rise

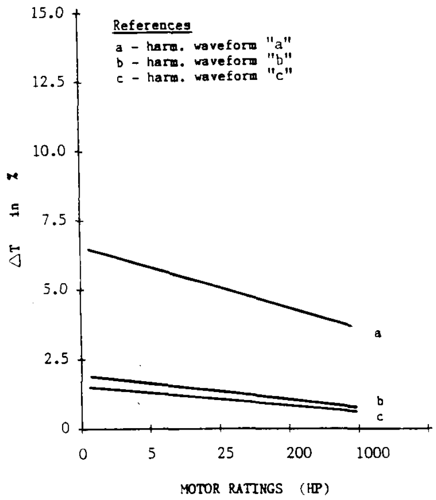

Fig. 5. Temperature increase on drip-proof radially cooled induction motor.

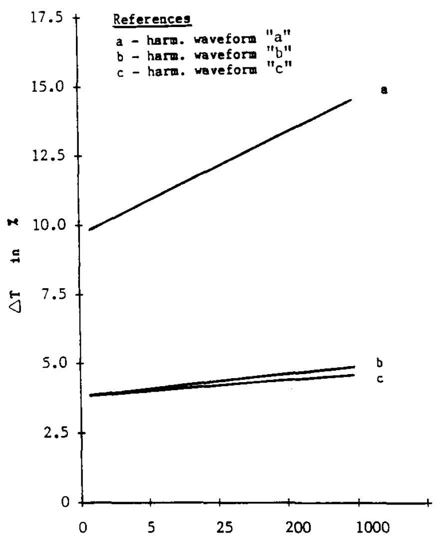

Fig. 6. Temperature increase on totally enclosed fan-cooled induction motor.

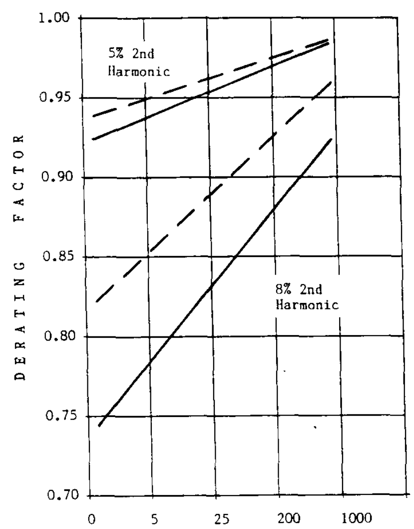

Fig. 7. Derating factors due to harmonic voltage distortion. Dotted lines: drip-proof radially cooled motor. Solid line: totally enclosed fan-cooled motor.

(less than 5%). It is also clear that, for drip-proof radially cooled machines, the percentage variation of temperature rise reduces as the size increases (negative slopes in Fig. 5), whereas it increases in totally enclosed fan-cooled machines (positive slopes in Fig. 6).

Based on the initial results, it is decided to calculate the derating due to second harmonics (case a) only. To determine the derating, a computer program reduces the output power of the machine until the temperature due to the abnormal condition is less than or equal to the normal operating temperature corresponding to class B insulation. The derating due to harmonic distortion is then calculated as

$$\text{derating}_H = 1 - \frac{P_{\text{out}\,H}}{P_{\text{out}}} \tag{29}$$

where $P_{\text{out}\,H}$ and P_{out} are the output power of the machine when supplied with nonsinusoidal and with sinusoidal voltage, respectively. The results are shown in Fig. 7.

When supplied with waveform a, machines of both types of enclosure and for all ratings would require derating at 5% harmonic distortion. Fig. 7 also shows the derating due to 8% second harmonic.

Conclusion

While, in the cases of harmonic waveforms b and c, 5% limitations on harmonic content is acceptable, results for case a are significant. We find that a restriction for the second harmonic should be included on the harmonic distortion limits established by IEEE Standard 519 [1], and derating in some cases should be considered for less than 5% harmonic distortion.

Drip-proof machines are found to be less affected by harmonic distortion than the totally enclosed machines. Efficiency plays a very important role in the degree of derating. Less efficient machines would require a higher derating. It is also clear that smaller machines (less than 5 hp) are affected more by the harmonics than are larger machines.

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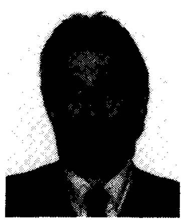

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