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FUNDAMENTAL CONCEPTS OF REGULATING DISTRIBUTION SYSTEM VOLTAGES

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FUNDAMENTAL CONCEPTS OF REGULATING DISTRIBUTION SYSTEM VOLTAGES

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Abstract **:** A power distribution system distributes power from a substation to the point of power utilization. Voltage drops occur naturally on distribution systems. It is designed to keep the voltages near the nominal values, and usually within a few percent limit. Besides load balancing, utilities commonly use both voltage regulators and shunt capacitors to maintain voltages within the design limits. This paper will address what factors to be considered when deciding whether regulators, capacitors, or load balancing or a combination is most appropriate to provide the needed voltage support.

I. INTRODUCTION

Distribution systems typically are not balanced due to a high concentration of single-phase loads. The load is also very dynamic and varies over time. New customers are added, while other customers may relocate, abandon, or change load at their facilities. All of these factors contribute to difficulty in controlling the distribution voltage within certain limits. The circuit model for a distribution system can be thought of as a series of impedances, with loads connected in between. The type of network often used to model a distribution circuit is a "ladder" network, and the repetitive nature of the calculations is most appropriate for computer analysis.

Performing accurate voltage calculations on a distribution system is a difficult task. An average feeder could easily serve 1000 or more customers, but modeling these many nodes for one feeder is impractical. Therefore, the individual customer loads must be grouped, and modeled as a reasonable number of nodes to obtain acceptable accuracy.

When an inductive current flows through an impedance, a voltage drop occurs. A typical model of the circuit conditions is given in Fig. 1(a), and the corresponding phasor diagram for one phase is shown in Fig. 1(b). The difference (ΔV) in voltage magnitude between V_1 and V_2 can be approximated for lagging power factor load conditions as (by ignoring the out-of phase voltage difference) :

$$\Delta V = I_w R + I_v X \qquad [1]$$

where, $Z = R + jX$ and $I = I_w - jI_v = I \angle -\theta$

A utility engineer has the following objectives when determining what equipment will be used to regulate the distribution system voltage and minimize the voltage drop :

- *To keep the voltage supplied at customers within a specified voltage range.*
- *To keep the average voltage supplied to customers centered near the nominal value and balanced between the phases, and*
- *To minimize the total cost of the distribution system.*

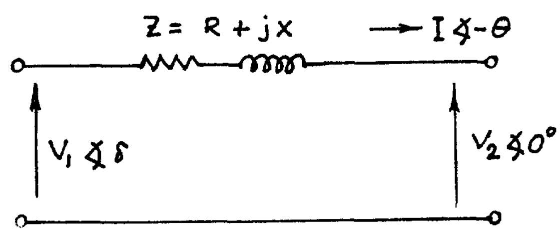

Fig. 1(a) Simplified 1-Phase Equivalent Circuit of a Distribution Feeder

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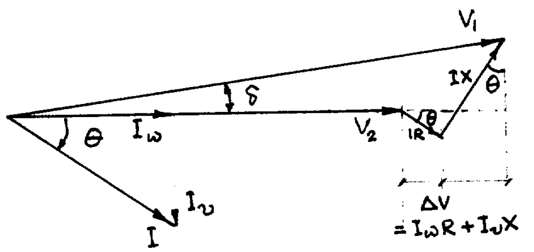


$$V_1 \approx V_2 + I_w R + I_v X$$

Fig. 1(b) Phasor Diagram Showing the Voltage-Current Relationship

II. ANSI STANDARD C84.1 [1]

Most utilities will attempt to maintain the voltage supplied to customers within a certain range specified by American National Standards Institute (ANSI) Standard C84.1 [1989].

The ANSI guideline is applied to both utilities and equipment manufacturers and refers to steady-state voltages only. The ANSI standard specifies both ranges : the service voltage, the utility supplies at the meter and the utilization voltage, voltage at the appliance. The range for the utilization voltage is wider than for the service voltage to allow for voltage drops within the premises wiring. The ANSI standard voltage ranges are included in Fig. 2 which is self explanatory.

The standard mentions *Voltage Range A* and *Voltage Range B*. The utility must strive to operate within Range A for the vast majority of the time which specifies a minimum service voltage of 0.95 per unit, and a maximum service voltage of 1.05 per unit (114 to 126 volts on a 120 volt base). Range B includes additional voltage drops on the utility system for a limited time, to allow for contingencies such as a forced outage or emergencies.

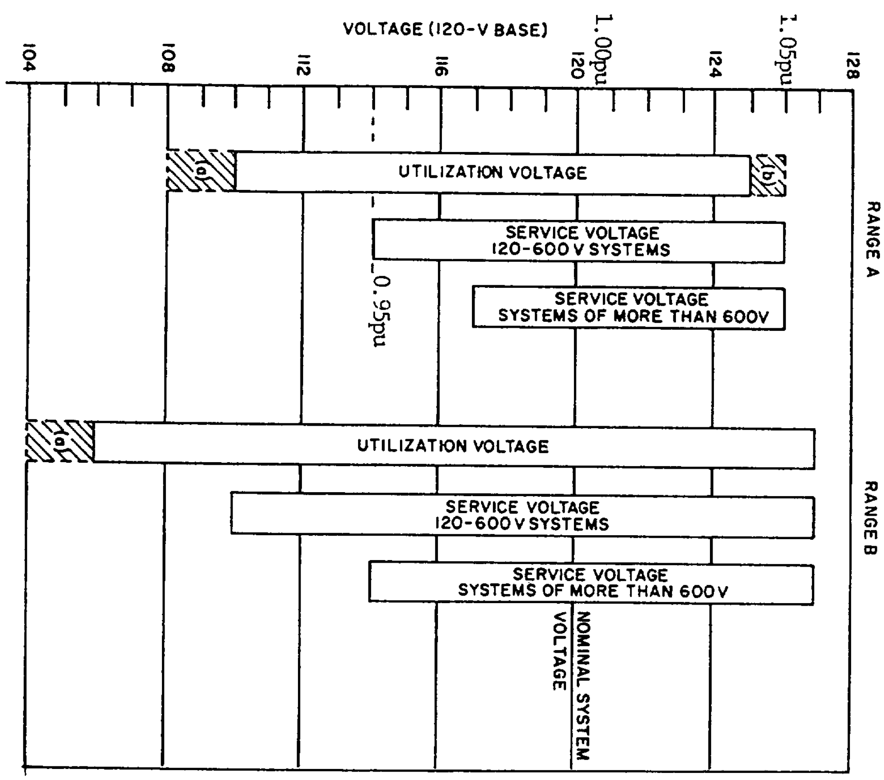


Fig. 2 ANSI Voltage Ranges [1]

III. OUTPUT VOLTAGE CONTROL AT THE SUBSTATION

Most utilities install some voltage regulating device at their substations, either through transformers load tap changer (LTC) or voltage regulators. The difference between a LTC and a voltage regulator is that the LTC is built into the transformer, by tapping the transformer winding in multiple locations and regulators are constructed as a separate unit from the transformer, using an autotransformer winding with many taps. The voltage regulators can be either three-phase or single-phase. Single-phase regulators are more common, because they can adjust each phase separately, and compensate for voltage unbalance.

It is typical for utilities to operate the substation slightly above the nominal voltage, to allow for drops on the distribution feeders, the transformers, and the service conductors.

IV. VOLTAGE CONTROL ON THE DISTRIBUTION SYSTEM

The challenge of distribution system voltage control is to limit the range of voltage variation such that the worst-case customer fed from the distribution system will meet the ANSI specification (or the utility's substitute criteria). A utility must allow for voltage drops through the secondary distribution transformers and the utility service conductors. These voltage drops represent a significant contribution to the total drop in the utility system. Most utilities assume a maximum transformer and service voltage drop of 4 to 5%.

If the substation voltage is set at 2% above nominal (or 1.02 pu), and the bottom of Range A is 5% below nominal (or 0.95 pu), this results in a 7% total operating range for the distribution and service systems. If a 4% allowance for the distribution transformer and service voltage drops is assumed, then the distribution system voltage drops must be limited to 3% to maintain service voltage within ANSI Range A. A graphical representation of the voltage range allowance is demonstrated in Fig. 3, which also includes ANSI Range B for comparison.

To maintain voltage drops on the distribution system within 3%, utilities must limit the length of the feeders, keep the system well-balanced, and correct the power factor. The longer the feeder, the more difficult it becomes to maintain distribution voltage in the necessary range. For distribution feeders,

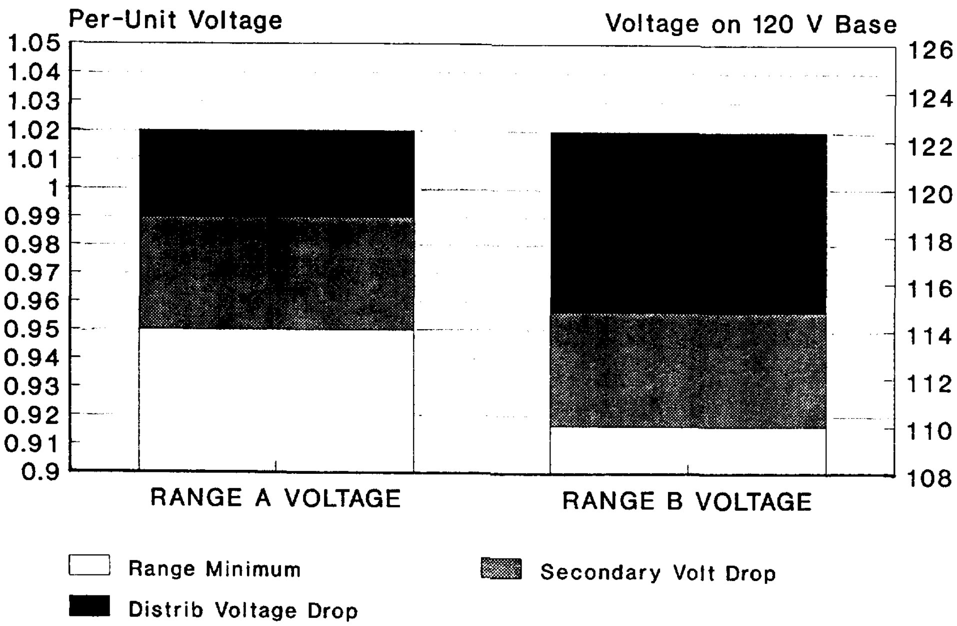


Fig. 3 Typical Division of Allowable Voltage Range

reaching several miles, the distribution voltage almost always needs to be raised with voltage regulators and/or shunt capacitors, besides reducing the unbalance loading between phases.

A. Effect of Unbalanced Load on Distribution Voltages

Distribution systems will typically have a great deal of single-phase load connected to them. Therefore, distribution systems are inherently unbalanced. If the phases are unequally loaded, losses are increased and the voltages become unbalanced.

When unbalanced loads are present, there is also current flowing on the system neutral (of a wye-connected system), creates a voltage drop on the neutral, resulting in a shift in the neutral potential. This neutral shift is toward the direction of the heavily loaded phase. The neutral shift lowers the magnitude of the heavily loaded phase voltage, and increases the magnitude of the lightly loaded phases, and exaggerates the effect of the voltage unbalance.

To avoid the problems caused through unbalanced voltages, it is important to balance the load among the three phases. A reasonable goal for load (current) balance to achieve in the utility industry is 10% or less and voltage balance in the 2% range.

B. Voltage Regulators

A voltage regulator is a multi-tap autotransformer which is built for the specific purpose of raising or lowering the distribution voltage. It is designed to provide precise control of the output voltage, even with large variations of the source voltage. Most regulators are single-phase devices. It is quite common to install three, single-phase regulators in a bank, to provide voltage regulation on a three-phase system. The regulators can be connected either grounded wye (most common), closed delta, or open delta.

A typical voltage regulator will provide ± 10% variation in voltage by using 16 steps of ± 5/8% taps. The variation in the output voltage is determined by the bandwidth setting on the control panel. The output voltage can be regulated to as close as ± 0.75%, even with the source voltage varying ± 10%. The tradeoff in a regulator is that the tighter the regulation band is desired on the output, the higher the number of tap changing operations will occur, resulting in a shorter lifetime for the regulator. Voltage regulators are designed for many operations, with typical designs exceeding 1,000,000 operations over the life of the regulator.An average number of operations per day could be about 70, which corresponds to over 25,000 operations in the course of a year.

A regulator is equally effective in solving high voltage or low voltage problems. The regulator contains a reversing switch, which can lower the voltage as well as raise it. When three single-phase regulators are applied on a three-phase system, they are also very well suited to solve voltage balance problems.

1) Sizing of Voltage Regulators : Voltage regulators are rated by either a maximum current rating, or the kVA rating. Allowance should also be made for the expected load growth. As an example, consider a voltage regulator rated at 7200 volts nominal and 69.4 amps, with a maximum voltage regulating range of 10%. The kVA rating of the regulator would be 0.10 x 7.2 kV x 69.4 amps = 50 kVA. Some regulators may safely be operated at higher currents than their rating, provided the range of voltage regulation is less than the maximum. The increase in current rating could be as much as 60% if the voltage regulation range is restricted to ± 5% instead of ± 10%.

2) Voltage Regulator and LTC Controls : Regulator controls have several adjustments, which are called voltage set, bandwidth, time delay, resistance setting, and reactance setting. The voltage set adjustment determines what the average voltage output will be, with the resistance and reactance settings ignored. The bandwidth adjustment determines a "dead band" around the voltage set point. As an example, say the voltage set point is at 122 volts (on a 120 volt base), and the bandwidth is set to 3 volts. The "dead band" is from 120.5 volts to 123.5 volts, which means that the LTC or the regulators will not raise the voltage unless it drops below 120.5 volts, and will not lower the voltage unless it exceeds 123.5 volts. The purpose of the bandwidth setting is to keep the number of tap changer operations to a reasonable value, and lengthen the life of the regulator. Normal bandwidth settings range from 1.5 to 6 volts.

The time delay setting specifies the duration which the voltage must stay outside the "dead band" before a tap changer operation is initiated. The major purpose of the time delay setting is to inhibit the tap changer from responding to short term voltage dips, such as those caused by the inrush currents from starting motors. Another use of the time delay settings is to co-ordinate the use of more than one LTC or regulators in series. Normal time delay settings range from 15 to 120 seconds.

The range for the resistive and reactive compensation adjustments is normally ± 24 volts. These resistance and reactance compensation settings are often used to increase the output voltage during peak loads, and reduce the output voltage during light load periods. The resistance control will adjust the voltage set point as a function of the kW load, and the reactance control will adjust the voltage as a function of kVAR load.

3) Determining Optimum Location for Voltage Regulators : Picking the proper location on the feeder is absolutely critical for voltage regulators. If a regulator is placed too close-in on the feeder, it may not provide enough voltage support to reach the end of the feeder. A regulator which is placed too far out on a feeder may cause too much of the load to be un-regulated, since the regulator affects voltage only on the output side of the regulator. Contingency configurations of the distribution system should also be considered when selecting the location for a regulator. If the regulator is installed on a tie feeder (and could be fed from a substation on either side of the regulator), the voltage profile should be considered when fed from either direction. Even if a regulator is installed on a radial (dead-ended) line, it may also be necessary to calculate voltage, if fed from an alternate source.

In the case of a line which has load tapped at discrete points along the line, it is best to place the regulator directly before the load is tapped. The regulator only affects voltage on the output side, so placing it before the load assures regulation for that load. Keeping the regulator close to the point where the load is tapped will assure that as much of the line voltage drop as possible is corrected.

C. Shunt Capacitors

The three reasons for installing capacitor banks in power distribution systems are *minimizing losses, releasing line and generation capacity and voltage support*. The most important reason is capacity release. The amount of capacitive kVAR installed on a system is usually determined by the economics of releasing generation and line capacity. Most utilities will have some sort of power factor "goal" to maintain at a specified location on their system during peak periods.

A common goal is to maintain a 98% lagging power factor on the sub-transmission system, but avoid operating at a leading power factor during light load periods. Because of VAR losses in the substation transformers, it is necessary to maintain the power factor during peak load at approximately 99% on the distribution system in order to achieve the 98% goal.

1) Voltage Effects of Capacitors : A shunt capacitor connected to the distribution system will cause a voltage rise. The approximate voltage (ΔV) rise can be calculated in one of two ways:

$$\Delta V = I_c * X_{\Omega} \text{ (volts)} \quad [2]$$

$$\Delta V = [\, Q_c / MVA_{base} \,] * X_{pu} \text{ (pu)}$$

where, Q_c = Capacitor Rating in MVAR (or kVAR)
I_c = Capacitor Current in amps
MVA_{base} = Base Rating in MVA (or kVA)
X = Line Reactance in (Ω) or in (pu)

Since the reactance of the line is a linear function of the length, the voltage rise which is caused by a capacitor is proportional to the capacitor kVAR, and the distance of line between the upstream voltage regulating device and the capacitor.

2) Sizing of Capacitor Banks : Individual capacitor can ranges from 25 kVAR to 500 kVAR for use on power distribution systems. Almost all capacitor installations are three-phase banks, but it is practical to install single phase units under certain conditions. Most banks are overhead, but padmount capacitor banks are also available for underground distribution feeders. However, they cost between 50% to 100% more per kVAR than overhead banks.

Larger banks can be made by paralleling several cans per phase. When more than one can per phase is used, often they will be individually fused.The banks can be connected in delta, grounded wye, or ungrounded wye. The grounded wye is the most popular connection because single-bushing units may be used, the fusing is simple, and overvoltages are not experienced if a unit fails. In areas where harmonics are a problem, the ungrounded wye connection is commonly used.

Most utilities have several standard sizes of distribution capacitor banks to choose from. The utility will stock standard sizes of capacitor banks which are unswitched (fixed) and other standard sizes which are switched. Because of the cost of the switching equipment (which includes switches, controller, PT and wiring), it is not economical to have small sizes of switched capacitor banks.

Of the total capacitor requirement, some can be on all the time, but some must be switched off during light load periods. The typical mix is somewhere around 50% of each. If the capacitors are not switched off during light load periods, problems with high voltage and instability could be encountered.

3) Determining Optimum Location for Capacitor Banks : Improving the power factor and consequently release of capacity is the most important reason for capacitor application, but the location does not affect generation or transmission line capacity release. As long as the capacitor is installed past a point where significant load is tapped (which is almost always the case whenever possible), the distribution capacity release is not usually a factor in location. Therefore, capacitor location is chosen based on either minimizing losses, or optimizing voltage support.

A number of papers have been written on methods to optimize capacitor placement based on minimizing losses. However, there has been very little literature on locating capacitors to optimize the voltage support. Often, the location picked for the capacitor bank (when the greatest need is voltage support) is based on a "trial and error" method of modeling the capacitor during peak load (to assure voltage does not fall too low), and during light load (to assure voltage does not become too high).

When determining the optimum location of a capacitor bank, the objective should be to locate the capacitor far enough downstream to keep the voltage above the minimum during peak load periods, but to stay upstream a sufficient distance to prevent the voltage from exceeding the maximum limit during light load periods. It is a big help if the bank is switched, because then it may be assumed to be off-line during the lightest load period. If the capacitor bank is switched, then the calculations for preventing high voltage should be done assuming the load is the least which could be expected during the time the switching controller would call for the bank to be switched on.

4) Switching Controls for Capacitor Banks: A fixed (unswitched) capacitor bank is much less expensive than a switched bank, so the capacitors are unswitched whenever it is practical. However, some of the capacitors on distribution systems must be switched. The larger capacitor banks often have some switching means so that they may be taken off line during light load periods.

Care must be used when selecting a capacitor switch, since the interruption of capacitive current is severe duty on a switch. Also, the capacitor switch will operate at least 2 times per day, often more, every day. Oil and vacuum switches are the best suited for capacitor switching. Voltage restrike is a possibility during capacitor switching, and should be considered when selecting switches.

There are two major reasons why capacitors banks are switched off line during light load. Capacitors cause a voltage rise. The transmission system produces VARs during light load periods, because the line charging current is greater than the reactive loss. Also, it is undesirable to operate the generation at a leading power factor for stability reasons.

Distribution capacitors can be switched by a number of different means. Common switch controllers operate on time, temperature, VARs, current, or voltage. Newer types of controllers sometimes combine two individual quantities, such as a time controller with a voltage override. Utilities which have some type of distribution automation system (SCADA) often use their system to switch capacitors. Switching the distribution capacitors by SCADA provides a great deal of flexibility, but is more costly to implement.

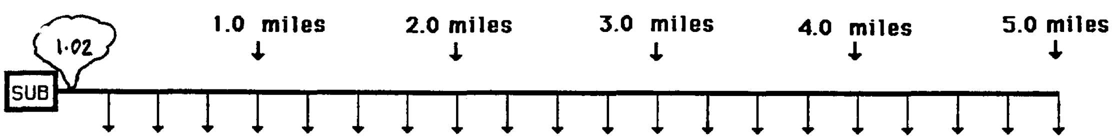
V. USING AN IDEAL FEEDER MODEL TO ILLUSTRATE CONCEPTS OF DISTRIBUTION VOLTAGE REGULATION

Real distribution feeders all have unbalanced currents and voltages, unevenly distributed loads, and laterals tapped off of the main lines. All of these complicating factors make it much more difficult to understand the concepts of voltage regulation. A computer model of an ideal feeder was developed which can be used to illustrate voltage regulation concepts. Voltage profile graphs of the ideal feeder are displayed to help understand the effects of capacitors, voltage regulators, and load balancing on distribution feeder voltages.

The ideal feeder shown in Fig. 4(a) is assumed to be a 5.0 mile feeder, constructed with #4/0 ACSR conductor and supplying a total load of 3.0 MW, at a 0.85 (lag) power factor. The load is assumed to be distributed evenly to 20 nodes, each spaced 0.25 miles apart, which results in a load of 150 kW per node. The load is assumed to be balanced exactly between the three phases. The voltage profile of the ideal feeder is shown in Fig. 4(b). Notice that the voltage drops at a faster rate near the beginning of the feeder and at a slower rate near the end of the feeder. In the example with evenly distributed load, 50% of the voltage drop occurs within the first 1.5 miles (30% of total feeder length), and 74% of the voltage drop occurs within the first half of the feeder.

A. Voltage Effects of Regulators

A line graph of the ideal feeder voltage (with and without the regulator) is shown in Fig. 5. It shows the effect of placing a regulator on the ideal feeder above at various locations (1, 2, 3 & 4 miles). The regulator was modeled with a bandwidth of 2.0 volts and zero compensation, set to keep the output



Load at each node is 150 kW + 92.96 kvar (pf = 0.85 lag)

Fig. 4(a) Ideal Feeder One-Line Diagram

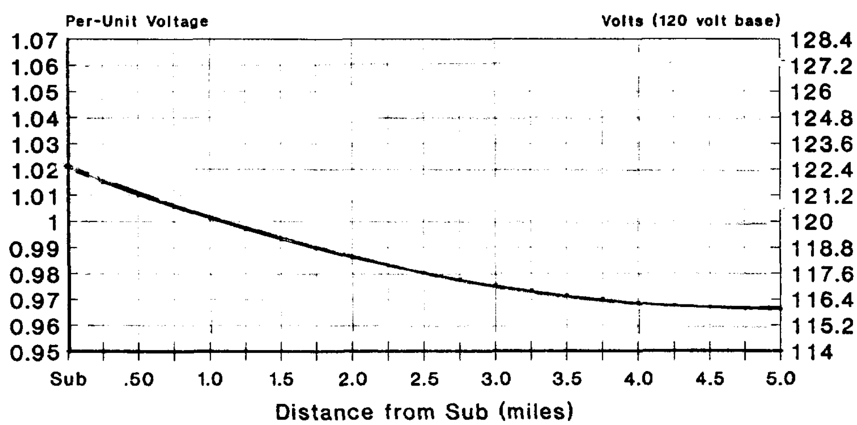


Fig. 4(b) Ideal Feeder Voltage Profile voltage at 1.02 pu. Notice that the voltage between the substation and the voltage regulator is unaffected by the regulator. The regulator provides a fixed voltage boost, so the voltage profile after the regulator parallels the voltage profile with no regulator. However, the regulator is a dynamic device, and the voltage boost from the regulator would be less during light loads, when the voltage drop is less.

Since the regulator provides voltage support to the system on the output side of the regulator only, placement of the regulator is very important.

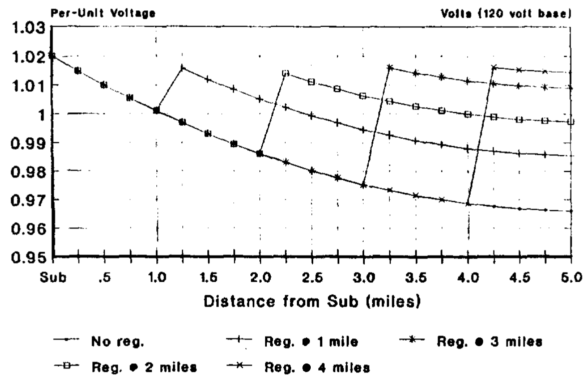


Fig. 5 Effect on Ideal Feeder Voltage Profile from Location of a Regulator Station

B. Determining Optimum Location for Voltage Regulators

Assuming uniform load distribution along the feeder, 50% of the voltage drop would occur in the first 30% of the feeder. Therefore, if one regulator is desired to split the voltage drop exactly, it would be placed at 30% * 5 miles = 1.5 miles from the substation. This example illustrates the need to calculate voltage drops on the feeder before a regulator is placed, rather than just installing regulators at evenly spaced intervals.

The location of the regulator bank is absolutely critical to providing a good voltage profile on the feeder. Notice how if the regulator is placed at one mile, the regulator provides only a small voltage boost. As a result, the voltage becomes low near the end of the feeder. However, if the regulator is placed at three miles from the substation, it provides a greater voltage boost, but the voltage before the regulator has dropped too low.

The number of regulator stations required on a line can be approximated from dividing the voltage drop on the line by the maximum variation allowed, subtracting one, and rounding up. As an example, assume a line with the load evenly distributed, and a voltage drop at the end of the line of 8%. To limit the voltage drop on this line to 3%, [(8/3) - 1] = 2 regulator stations can accomplish this design. Spacing of the regulators can be approximated by dividing the feeder into segments which would have voltage drops that are approximately equal.

In the case of the ideal feeder, placing one regulator at the 1.5 mile location would result in the flattest voltage profile as shown in Fig. 6(a). If two regulators are proposed for this ideal feeder, the optimum locations to place them would be at 1.0 and 2.25 miles in order to split the voltage drop in thirds. Fig. 6(b) depicts the voltage profile of the ideal feeder with regulator banks at 1.0 and 2.25 miles.

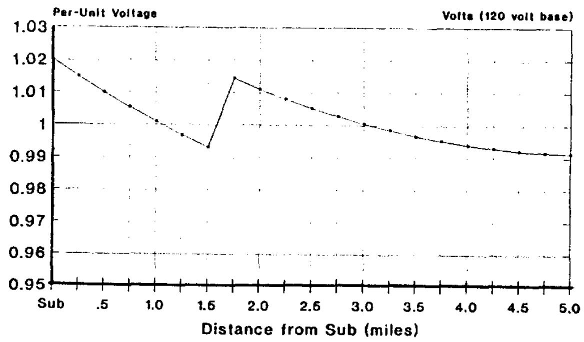


Fig. 6(a) Ideal Feeder Voltage Profile with One Regulator at 1.5 Miles

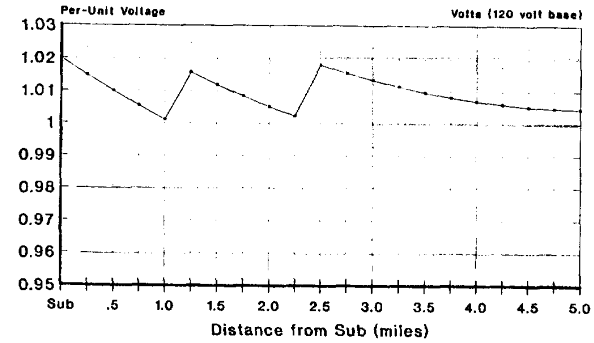


Fig. 6(b) Ideal Feeder Voltage Profile with Two Regulators at 1.0 and 2.25 Miles

C. Modeling Capacitor Banks on the Ideal Feeder

The effect of capacitor banks on the ideal feeder voltage profile is also investigated. Fig. 7 illustrates the effect of a 2400 kVAR capacitor bank on our ideal feeder and depicts the normal voltage profile of the feeder with no capacitor bank, and the voltage profile due to the capacitor only (with no load) and the net effect of the load and the capacitor (in the middle).

Notice that the voltage rise due to the capacitor is a straight-line function of the distance from the substation, until the point where the capacitor is located. After the capacitor node, there is no additional voltage rise. The net result of the load and capacitor combined can be found by adding the voltage rise effect of the capacitor to the voltage profile of the feeder without the capacitor.

The voltage drops (due to load) can be compensated with a capacitor bank. The size of the capacitor and the location can be chosen such that at a certain point on the feeder and a given loading, the voltage rise from the capacitor will exactly balance the voltage drop due to the load. The ideal feeder modeled shows a capacitor size and location which is chosen to very nearly balance the voltage drop from the load.

The choice of a 2400 kVAR capacitor bank and the location at 3.5 miles results in a voltage profile that is relatively flat. Since the feeder loading (and therefore the feeder voltage drop) is dynamic, the size and location of the capacitor must be chosen as a compromise between peak load and light load conditions. However, a capacitor bank may be switched, & the switching is entirely off or on. It is important to remember that a capacitor bank is not a dynamic device like a regulator.

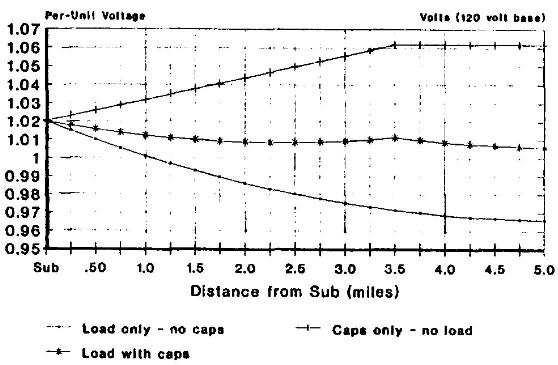


Fig. 7 Ideal Feeder Voltage Profile with Capacitor Bank at 3.5 Miles from Substation

D. Optimum Size and Location of Capacitors

Fig. 8 illustrates how the location of the capacitor bank affects the voltage profile. There are four separate voltage profiles plotted, representing 2400 kVAR capacitor banks placed at 1.0, 2.5, 3.5, and 4.75 miles. Notice that all the plots overlay for the first mile, but diverge when a capacitor location is reached. Moving the capacitor farther out on the feeder does not affect the voltage profile near the beginning of the feeder, but it does affect the voltage profile near the end of the feeder.

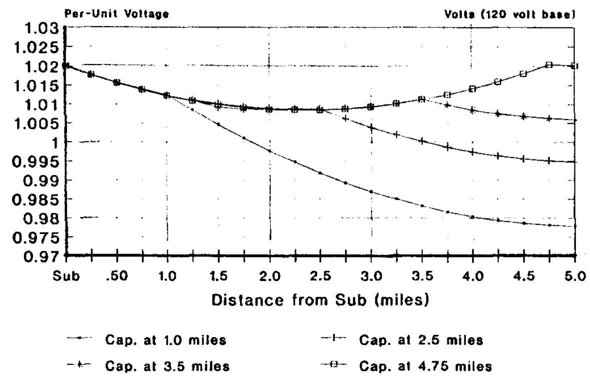


Fig. 8 Effects on Ideal Feeder Voltage Profile From Location of a 2400 kVAR Capacitor Bank

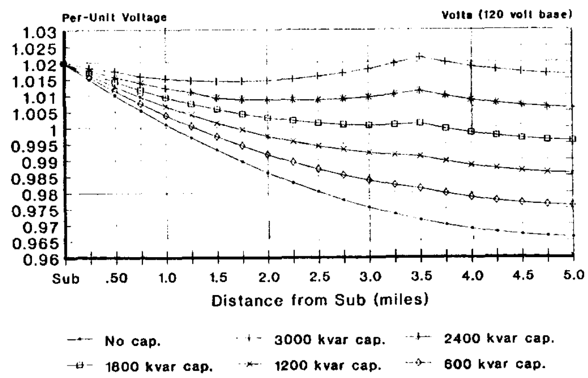


Fig. 9 Effects on Ideal Feeder Voltage Profile From Varying Size of Capacitor Bank @ 3.5 Miles

Next, the location of the capacitor bank was held constant at 3.5 miles, and the size of the bank was varied. Fig. 9 shows the change in the voltage profile as the capacitor bank is increased from 0 to 3000 kVAR in increments of 600 kVAR. The size affects voltage along the entire feeder length, while location determines how far the effect of the capacitor bank extends.

Capacitors also reduces losses in the feeder. Fig. 10 shows how the total kW losses on the ideal feeder vary as a function of the capacitor location. The diagrams demonstrate that there is a location for the capacitor bank which minimizes the losses on the feeder. The location for minimum losses using one capacitor bank is at 2.0 miles for a 2400 kVAR bank or 3.75 miles for a 1200 kVAR bank.

Notice that the shape of the losses curve is similar to a parabola. It is obvious that placing the capacitor at the very beginning or the very end of the feeder is a poor idea for minimizing losses. Consider the location of the 1200 kVAR capacitor bank. Because of the parabolic shape of the losses curve, the difference between locating the capacitor bank at the optimum location (at 3.75 miles) and locating it one mile each direction (at 2.75 miles or at 4.75 miles) is only 3 kW.

VI. CONCLUSIONS

In summary, several lessons are learned from modeling the ideal feeder. The rate at which the voltage drops is highest near the beginning of the feeder, where the current is greatest. The size of a capacitor bank affects the magnitude of the voltage rise on the entire feeder, while the location determines how far along the feeder the capacitor bank affects the voltage. Finally, a voltage regulator raises the voltage output of the regulator to a value determined by the settings of the regulator. Voltage regulators are the most versatile and effective means to control the distribution voltage. However, they are also costly solution. The economic benefits of power factor correction are great, and those benefits can usually justify capacitors applications.

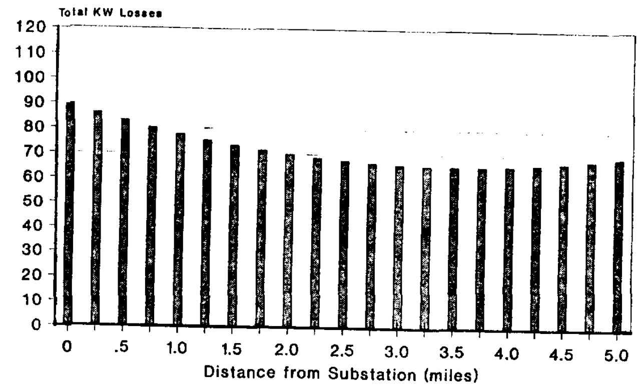


Fig. 10(a) kW Losses on Ideal Feeder as a Function of Distance for a 1200 kVAR Capacitor Bank

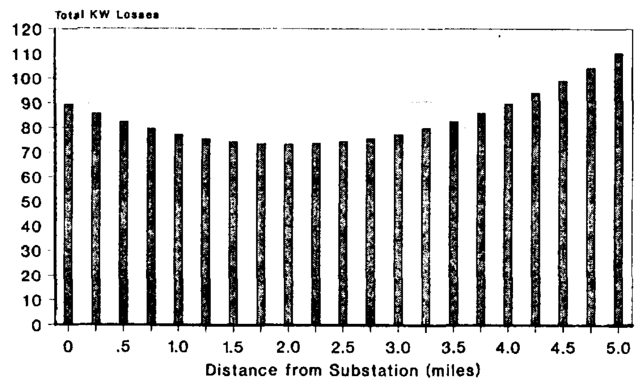


Fig. 10(b) kW Losses on Ideal Feeder as a Function of Distance for a 2400 kVAR Capacitor Bank

VII. REFERENCES

[1] *American National Standards for Electric Power Systems and Equipment - Voltage Ratings (60 Hz)*, C.84.1 - 1982.

[2] *Distribution Systems Engineering Reference Book*, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania, 1965.

[3] M. M. A. Salama, A. Y. Chikhani, and R. Hackman, "Control of Reactive Power in Distribution Systems with an End-Load and Fixed Load Condition," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, pp. 2779-2788, October 1985.

[4] S. L. Larson, "A Comparison of Methods for Regulating Distribution System Voltages," *M.S. Report*, University of Colorado at Boulder, 1991.