A Better Understanding of Load and Loss Factors

Keith Malmedal, P.E.
Member IEEE
Senior Engineer/Project Manager
NEI Electric Power Engineering
Arvada, Colorado 80001
kmalmedal@neiengineering.com

P.K. Sen, Ph. D, P.E.,
Senior Member IEEE
Professor of Engineering
Colorado School of Mines
Golden, Colorado 80401
psen@mines.edu

***Abstract*- To minimize the necessary calculations when performing loss studies, utilities often measure load factors, try to determine the loss factor, and use the estimated loss factor to determine system losses. This paper examines the relationship between load and loss factors, and discusses the validity of common methods used to determine losses using load and loss factors.**

I. INTRODUCTION

Loss studies are often performed on a customer, primary or secondary feeder, transmission line, piece of equipment, or a complete system to determine the amount of energy lost during some period of time. Determining total system energy losses is often simply done by subtracting metered energy sold from metered energy purchased [1]. However, a loss study my also attempt to determine how much of this energy is being lost on different parts of the system, and how much is being lost through theft or inaccurate metering. This information is valuable in economic planning for future growth, or for use in a loss reduction program to improve utility efficiency.

In the United States 9% of gross generation is lost each year in the distribution and transmission system [2] 4% of which is accounted for by the transmission system [3]. Recommended distribution system losses are between 5%-11% of their input energy depending upon how urban the distribution system location is [1].

Losses in a system fall into three categories:

1. Constant losses due to transformer magnetization losses
2. I^2R losses that will vary with load
3. Losses due to theft and metering error

Constant losses may be computed if the number of transformers and the tested characteristics of those transformers are known. The I^2R losses are more difficult to determine since they vary at every point in the system due to variations in conductor resistance and current. Theft and metering error losses may be determined if the other two types of losses may be calculated and the total system loss is known by subtracting the constant and I^2R losses from the total system losses.

This paper will be primarily concerned with computing the I^2R losses in the system. These losses may only be accurately computed by determining the average load on each component during each hour and doing a separate loss calculation for each hour during the time period in question. Due to the computational complexity of this approach, a simplified method of finding system energy losses using load and loss factors is often attempted.

II. LOAD AND LOSS FACTORS

Load factor is defined as:

$$\text{Load Factor} = \frac{\text{Load}_{avg}}{\text{Load}_{peak}} \quad \textbf{(1)}$$

Where:

Load_{avg} = Average load (kW) over the period in question
Load_{peak} = Peak load (kW) over the period in question

Loss factor is similarly defined as:

$$\text{Loss Factor} = \frac{\text{Loss}_{avg}}{\text{Loss}_{peak}} \quad \textbf{(2)}$$

Where:

Loss_{avg} = Average loss (kW) over the period in question
Loss_{peak} = Peak loss (kW) over the period in question

Load factors may be determined using measurements which produce a load profile on the component in question. One such load profile is shown in Figure 1. This Figure is the measured load profile for a 6.6 kV distribution feeder for a 24 hour period. A similar load profile can be measured for any period in question. Often, this method of determining system losses is applied on a monthly basis [4].

The average load for the feeder shown in Figure 1 is 1.85MW. The peak load is 3.2MW. This results in a load factor of approximately 0.578.

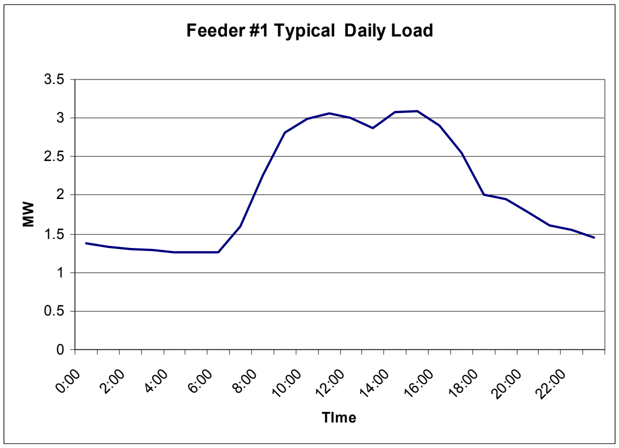

Figure 1. Feeder #1 typical load profile

If the resistance of the feeder in Figure 1 were known, the peak load could be used to determine the peak loss on this feeder. If the loss factor were known, then multiplying the peak loss by the loss factor using equation [2] should return the average loss on this feeder which may be multiplied by the number of hours in the period under study to determine the total energy loss on this feeder over this period.

This is a much simplified method since it takes only one calculation using the peak load and the feeder resistance to calculate only the peak loss instead of doing a separate loss calculation for each hour. However, this method assumes that the loss factor can be determined from the load profile.

It also must be emphasized that the losses calculated using this method are only those that are a function of the load (I^2R losses). Constant losses, which do not vary with load, cannot be computed using the loss factor. Constant losses must be calculated independently and added in to the result to get total losses.

III. CALCULATING THE LOSS FACTOR

Equation (3) is suggested to calculate the loss factor using only information contained in the load profile [1][4].

$$\text{Loss Factor} = a(\text{Load Factor}) + (1-a)(\text{Load Factor})^2 \quad \textbf{(3)}$$

Equation (3) shows that in addition to the load factor the value of "a" must be determined. Equation (3) may be solved for "a" and re-arranged as:

$$a = \frac{\text{Loss Factor} - (\text{Load Factor})^2}{\text{Load Factor} - (\text{Load Factor})^2} \quad \textbf{(4)}$$

Assuming constant power factor, a balanced system, and constant voltage, and using equations (1) and (2):

$$\text{Load Factor} = \frac{I_{avg}}{I_{peak}} \quad \textbf{(5)}$$

$$\text{Loss Factor} = \frac{(I^2)_{avg} R}{(I^2)_{peak} R} = \frac{(I^2)_{avg}}{I^2_{peak}} \quad \textbf{(6)}$$

I_{avg} = Current averaged over the period
I_{peak} = Peak currrent during the period
$(I^2)_{avg}$ = Current Squared averaged over the period
I^2_{peak} = Peak current squared during the period
R = System resistance

Combining equations (5) and (6) with equation (4) results equation (7).

$$a = \frac{(I^2)_{avg} - (I_{avg})^2}{I_{peak} I_{avg} - (I_{avg})^2} \quad \textbf{(7)}$$

From equation (3) it can be seen that if a=0 then the Loss Factor = (Load Factor)^2. Equation (7) shows that a=0 only if:

$$(I^2)_{avg} - (I_{avg})^2 = 0 \quad \text{so} \quad (I^2)_{avg} = (I_{avg})^2 \quad \textbf{(8)}$$

The average of the current squared may equal the square of the average current only when the current is constant. So it may be seen that a=0, and (Loss Factor) = (Load Factor)^2 only if the load is constant. Also, in a constant load condition (Peak Load) = (Average Load) and (Peak Loss) = (Average Loss) so both load and loss factors are 1.

Equation (3) shows that if a=1 then (Loss Factor)=(Load Factor). If both load and loss factors equal 1, then "a" may equal 0 as already shown or "a" may equal 1. In either case the load is constant if both the load and loss factors are 1. For a constant load, where both load and loss factors equal 1, then both of the following are true:

Loss Factor = Load Factor
Loss Factor = (Load Factor)^2

The only time "a" may equal 0 and the Loss Factor = (Load Factor)^2 is in the case of a constant load. However, while a constant load will also result in a=1, there is one other case where "a" may equal 1 and the Loss Factor may equal the load factor.

If we consider a total time interval which we divide into several samples of time length "t" totaling T samples, and take current sample I_n which is the average current during time intervals "t", and assuming that the current I_n is nearly constant over the time "t" and I_{peak} is the peak current during all of time interval T, then:

$$\text{Average Load} = I_{avg} = \frac{\sum_{n=0}^{T} I_n t}{T} \quad \textbf{(9)}$$

$$\text{Average Loss} = \frac{\sum_{n=0}^{T} I_n^2 R t}{T} \quad \textbf{(10)}$$

$$\text{Load Factor} = \frac{\sum_{n=0}^{T} I_n t}{T I_{peak}} = \frac{t}{T}\sum_{n=0}^{T} \frac{I_n}{I_{peak}} \quad \textbf{(11)}$$

$$\text{Loss Factor} = \frac{\sum_{n=0}^{T} I_n^2 R t}{T I_{peak}^2 R} = \frac{t}{T}\sum_{n=0}^{T} \frac{I_n^2}{I_{peak}^2} \quad \textbf{(12)}$$

Comparing equations (11) and (12) term-by-term, for the Loss factor to equal the load factor:

$$\sum_{n=0}^{T} \frac{I_n^2}{I_{peak}^2} = \sum_{n=0}^{T} \frac{I_n}{I_{peak}}$$

and

$$\frac{I_n^2}{I_{peak}^2} = \frac{I_n}{I_{peak}} \quad \textbf{(13)}$$

so

$$I_n^2 = I_n I_{peak}$$

Equation (13) is true for a constant current where $I_{peak}=I_n$ for each term and it is also true for a bimodal load profile where the current is either zero or always equal to the same value. An example of this is shown if Figure 2 where the load is either zero or 250kW.

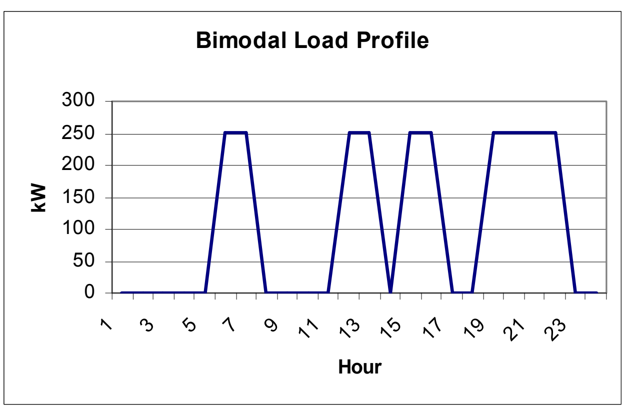

Figure 2. Bimodal Load Profile

Assuming a component resistance of 5 ohms the load profile in Figure 2 results in a load and loss factor of 0.21

If we ignore the cases where a=0 or 1, and further compare the Load and Loss Factors using equation (13) we see that:

since

$$I_n^2 \le I_n I_{peak}$$

then

$$\frac{I_n^2}{I_{peak}^2} \le \frac{I_n}{I_{peak}}$$

and

$$\text{Loss Factor} \le \text{Load Factor}$$

Since the loss factor is always less than or equal to the load factor, and the loss and load factors must always be less than 1, then we can conclude from equation (4) that:

$$a \le 1 \quad \textbf{(14)}$$

Using equations (5) and (6):

$$\text{Load Factor}^2 = \frac{(I_{avg})^2}{I_{peak}^2}$$

and

$$\text{Loss Factor} = \frac{(I^2)_{avg}}{I_{peak}^2} \quad \textbf{(15)}$$

$$\frac{(I_{avg})^2}{\text{Load Factor}^2} = \frac{(I^2)_{avg}}{\text{Loss Factor}}$$

so

$$\text{Loss Factor} = \frac{(I^2)_{avg}}{(I_{avg})^2}\text{Load Factor}^2$$

Since the average of the I^2 will be equal to $(I^2)_{avg}$ if the load is constant, and will be greater than $(I^2)_{avg}$ in all other cases, then the following may be concluded:

$$\text{Loss factor} \ge \text{Load Factor}^2 \quad \textbf{(16)}$$

The following characteristics of "a" have been determined:

1. If a=0
   a. Loss Factor = Load Factor^2
   b. Load Factor=1
   c. Loss Factor = 1
   d. The load is constant
2. As "a" approaches 0 the load becomes more nearly constant
3. If a=1
   a. Loss Factor=Load Factor
   b. The load may be constant or bimodal
4. As "a" approaches 1 the load becomes more nearly bimodal where one mode=0
5. For most practical cases 0 < a < 1

Additionally it has been determined that:

$$\text{Loss Factor} \le \text{Load Factor}$$

$$\text{Loss Factor} \ge (\text{Load Factor})^2$$

IV. DETERMINATION OF "a"

Figure 3 shows a plot of every possible value of "a". This plot was made from the data contained in Table 1. This table includes data for the two special cases a=1 and a=0 for purposes of interpolation. However, the values of load and loss factors for these endpoints of "a" have already been discussed.

For any value of Load Factor a range of Loss Factor values are possible. This range of Loss Factors becomes smaller as the value of the Load Factor approaches 0 or 1. For example, at a Load Factor of 0.1 the Loss Factor may range between 0.01 and 0.1. However, at a Load Factor of 0.5 the range of Loss Factors becomes greatest ranging between 0.25 and 0.5

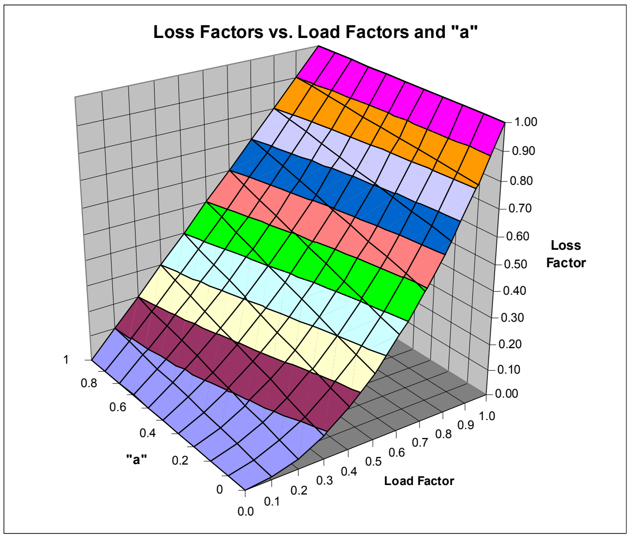

Figure 3. Load and loss factors and "a"

Loss Factor											
Load Factor	a										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	0.01	0.02	0.03	0.04	0.05	0.06	0.06	0.07	0.08	0.09	0.10
0.2	0.04	0.06	0.07	0.09	0.10	0.12	0.14	0.15	0.17	0.18	0.20
0.3	0.09	0.11	0.13	0.15	0.17	0.20	0.22	0.24	0.26	0.28	0.30
0.4	0.16	0.18	0.21	0.23	0.26	0.28	0.30	0.33	0.35	0.38	0.40
0.5	0.25	0.28	0.30	0.33	0.35	0.38	0.40	0.43	0.45	0.48	0.50
0.6	0.36	0.38	0.41	0.43	0.46	0.48	0.50	0.53	0.55	0.58	0.60
0.7	0.49	0.51	0.53	0.55	0.57	0.60	0.62	0.64	0.66	0.68	0.70
0.8	0.64	0.66	0.67	0.69	0.70	0.72	0.74	0.75	0.77	0.78	0.80
0.9	0.81	0.82	0.83	0.84	0.85	0.86	0.86	0.87	0.88	0.89	0.90
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 1

Since a range of loss factors may be determined whenever a load factor is known, then if a peak loss is calculated on a system a range of average losses may be calculated using the data in Table 1 and Figure 3.

For example: If a load factor of 0.5 was measured, and the peak losses on a system were calculated as 100kW during a 30 day period, then the loss factor would range between 0.25 and 0.5. This would mean that the average loss on the system during this period was between 250kW and 500kW, resulting in a 30 day energy loss (assuming 720 hours in the month) of between 18,000 and 36,000kWh.

In an attempt to more closely determine the losses on a system a value of "a" is sometimes chosen. The suggested value of "a" varies with the source consulted. One source [1] suggests choosing a=0.16 based on "various load studies". Another source [4] suggests that a=0.3 is a reasonable value to use in finding the "30-minute, monthly, kW Loss Factor."

V. EMPIRICAL DETERMINATION OF "a"

In the course of performing a load study for the transmission and distribution system of the country of Belize, measurements were taken to determine the load profile several distribution feeders and hourly data was taken on various parts of the distribution system to calculate I^2R losses. This empirically derived data was then used to calculate load and loss factors and determine the actual value of the constant "a".

This calculation was done on a monthly basis, and it was found that the value of "a" was not constant but varied during the year. These feeders ranged in voltage between 6.6kV and 22kV. Peak loads varied from 1 to 6MW. Table 2 shows the monthly data on two of these feeders.

	Feeder #1			Feeder #2		
Month	**Load Factor**	**Loss Factor**	**a**	**Load Factor**	**Loss Factor**	**a**
Jan	0.61	0.41	0.15	0.43	0.24	0.23
Feb	0.36	0.15	0.06	0.44	0.25	0.22
March	0.60	0.39	0.13	0.42	0.24	0.26
April	0.62	0.42	0.16	0.39	0.24	0.36
May	0.67	0.48	0.12	0.39	0.24	0.36
June	0.69	0.50	0.13	0.42	0.27	0.36
July	0.67	0.48	0.18	0.40	0.25	0.37
August	0.67	0.48	0.12	0.40	0.24	0.33
Sept	0.72	0.55	0.12	0.43	0.28	0.37
Oct	0.71	0.53	0.12	0.42	0.26	0.34
Nov	0.61	0.40	0.09	0.43	0.25	0.29
Dec	0.64	0.44	0.13	0.40	0.22	0.26
		AVERAGE	0.13			0.31

Table 2

Table 3 shows the values of Load and Loss Factors averaged over one year for all the feeders measured and their average calculated "a".

Feeder	**Load Factor**	**Loss Factor**	**a**
#1	0.63	0.44	0.13
#2	0.42	0.25	0.31
#3	0.54	0.33	0.14
#4	0.66	0.51	0.32
#5	0.63	0.44	0.19
#6	0.56	0.38	0.26
#7	0.60	0.41	0.11
#8	0.52	0.32	0.21
#9	0.70	0.52	0.12
#10	0.63	0.44	0.19
#11	0.62	0.44	0.24
#12	0.67	0.49	0.17
Average	0.60	0.41	0.20

Table 3

A transmission line load profile was also used to calculate the value of "a". Due to the diversity on a transmission line the load would be expected to be more constant than a distribution line. Therefore it would be expected that "a" would be closer to 0 than would be expected for a transmission line. The load profile for the line is shown in Figure 4.

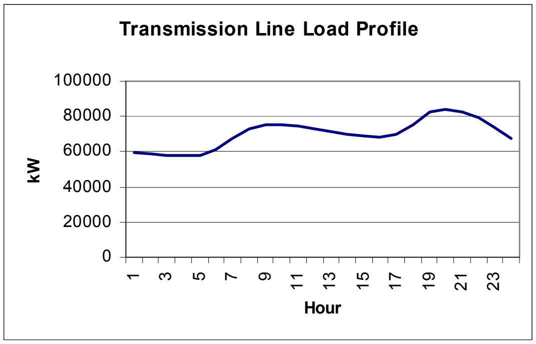

Figure 4. Transmission line load profile

The following values were calculated:

Load Factor = 0.83
Loss Factor = 0.70
a=0.06

It may be seen from the collected data that for a distribution system feeder or a transmission line the arbitrary use of 0.16 or 0.30 for "a" will result in some error.

VI. Calculations for More Complex Systems

The equations and calculations shown thus far are valid for single load profiles on single system components (one current applied to only one resistance). Can these same methods be applied to more complex systems, such as a distribution line with several loads at several locations on the line?

Consider the example in Figure 5. This is a distribution feeder with three loads separated by three line segments with resistances R_1, R_2, and R_3.

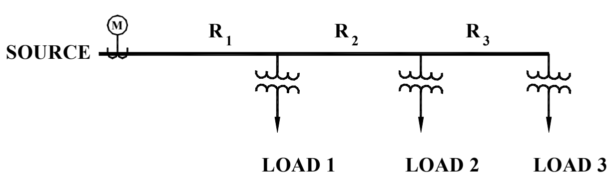

Figure 5. Distribution feeder with three loads

It may be seen that it is possible to produce load profiles for these loads that will invalidate the assumptions made for Load and Loss Factors and "a" which were used for single components. For example, assuming that metering was done only where shown, and applying the following load profiles to line segments where R_1=R_2=R_3= 1 Ohm it may be seen that the assumptions we have arrived at for the value of "a" are no longer valid.

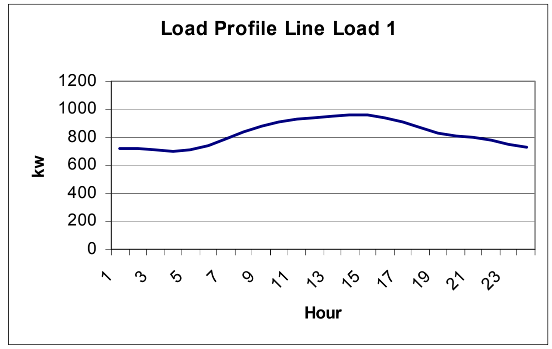

Figure 6. Load profile load 1

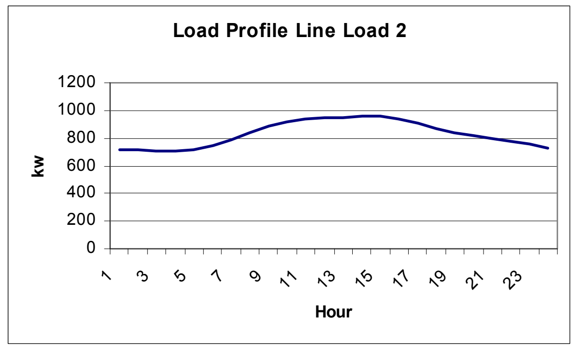

Figure 7. Load profile load 2

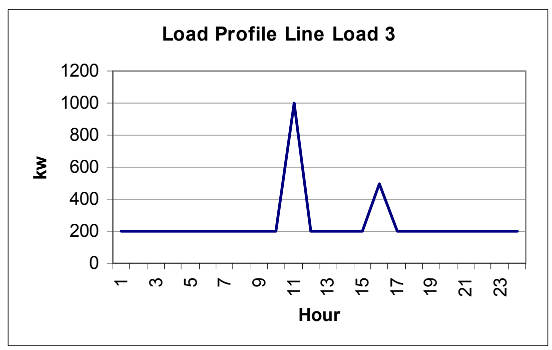

Figure 8. Load profile line 3

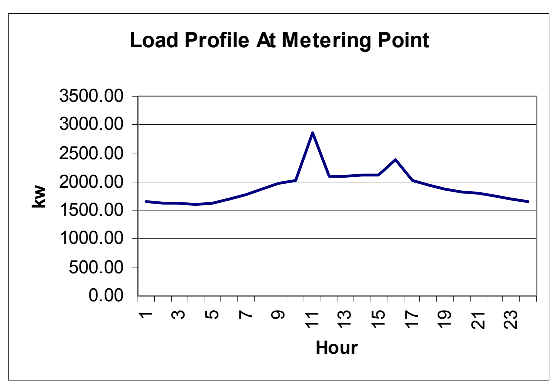

Figure 9. Load profile at metering point

The values in Table 4 may be computed for each line segment and for the system as a whole as measured at the metering point shown.

	Avg Load (kW)	Peak Load (kW)	Avg Loss (kW)	Peak Loss (kW)	Load Factor	Loss Factor	a
Seg. 1	1903.99	2868.00	23.83	52.90	0.66	0.45	0.04
Seg. 2	1074.91	1934.00	7.73	24.05	0.56	0.32	0.05
Seg. 3	245.83	1000.00	0.57	6.43	0.25	0.09	0.15
Total Line	1903.99	2868.00	32.13	83.38	0.66	0.39	-0.25

It may be seen that for the total line the Loss Factor no longer exceeds the Load Factor^2 and the computed value of "a" is negative. Arbitrarily choosing a=0.3 for this example would have produced a loss factor of 0.5 and a calculated average loss of 41.7 kW instead of the actual 32.13kW. This would have been an error of nearly 30%.

VII.CONCLUSIONS

The method of finding the load profile, calculating the Load Factor, using equation (3) to determine the Loss Factor, and finding average losses from the Loss Factor and the calculated peak loss is valid for single load profiles applied to single system components. By choosing all possible values of "a" for the measured load profile a range of average losses can be determined. While a value of "a" is often arbitrarily chosen, the values commonly used will result in some error as shown by the actual examples included in this paper. Also, the values of "a" should be expected to vary from month to month. The values of "a" for a transmission system should be expected to be far less than for a distribution feeder due to the larger diversity on the transmission line.

The application of this method beyond a single component may produce results which are very inaccurate. It may be difficult or impossible to determine the value of "a" or even a range of values for "a" if the system is complex and multi-branched with several load profiles. The values of "a" can vary far more then expected and can become negative under some conditions. More research needs to be done to determine how valid this method is when used on a complete multi-branched system before depending upon it to accurately determine losses in a complex system.

REFERENCES

[1] *Distribution System Loss Management Manual*, National Rural Electric Cooperative Association, Washington D.C. 1991

[2] Energy Flow 2004, Diagram 5, **www.eia.doe.gov/emeu/aer/pdf/pages/sec8_3.pdf**

[3] Bulk Transmission System Loss Analysis, EPRI Reports EL-6814-V1 & EL-6814-V2, 1990.

[4] Westinghouse Electric Corporation, *Distribution Systems, Electric Utility Engineers, Electric Utility Engineering Reference Book, Vol. 3* East Pittsburg, PA; Westinghouse Electric Corp. 1959.

[5] Perry G. Brittain, "Distribution Transformer Loading by the Load Factor-Computer Method," Proceedings of the American Power Conference, 1957

[6] B. M. Gallaher, "Problems of Power Supply for Three-Phase Packaged Air-Conditioner Units," Edison Electric Institute Bulletin, Volume 21, January 1953, pp 11-15

[7] R.H. Sarikas and H.B. Thacker, "Distribution System Load Characteristics and Their Use in Planning and Design," AIEE Transactions, Vol. 76():564-73, August 1957

Keith Malmedal received his BSEET degree from Metropolitan State College of Denver in 1995, a MSEE degree (Power) and a MSCE degree (Structures) from the University of Colorado at Denver in 1998 and 2002, respectively and is presently a Ph.D. candidate at Colorado School of Mines in Engineering Systems (Electric Power Emphasis). He has over fifteen years experience in electrical power design and is presently a senior design engineer and project manager at NEI Electric Power Engineering, Arvada, Colorado, specializing in all aspects of power system design. Mr. Malmedal is a registered professional engineer in 14 states

Pankaj K. (PK) Sen received his BSEE degree (with honors) from Jadavpur University, Calcutta, India, and the M.Eng. and Ph.D. degrees in electrical engineering from the Technical University of Nova Scotia (Dalhousie University), Halifax, NS, Canada. He is currently Professor of Engineering and Director of the Power Systems Engineering Research Center at Colorado School of Mines in Golden, Colorado. His research interests include application problems in electric machines, power systems, and power engineering education. He has published more than 70 articles in various archival journals and converence proceedings. Dr. Sen is a registered professional engineer in the State of Colorado.