EXPERIMENTAL EVIDENCE REJECTING A COMMON METHOD FOR FINDING SOIL THERMAL STABILITY

Keith Malmedal, PhD, PE, P.Eng
Senior Member IEEE
NEI Electric Power Engineering
P.O. Box 1265
Arvada, CO 80001
USA
kmalmedal@neiengineering.com

Carson Bates, MSEE, PE
Graduate Student Member IEEE
NEI Electric Power Engineering
P.O. Box 1265
Arvada, CO 80001
USA
cbates@neiengineering.com

David Cain, AAS
Electrical Engineering Intern
NEI Electric Power Engineering
P.O. Box 1265
Arvada, CO 80001
USA
dcain@neiengineering.com

***Abstract* – The heat generated by underground cables has been known to cause the soil around the cables to dry, increasing its thermal resistivity and potentially causing the cables to overheat. The ability of soil to maintain a constant resistivity while being subjected to a heat source is known as its "thermal stability". A method using the Law of Times has often been recommended to find soil stability. To test whether this method can accurately predict soil thermal stability an experiment was performed that tested the hypothesis inherent in the Law of Times that the diameter of the heat source affects the drying time of the soil surrounding it. This paper reports the results of that experiment and includes the statistical analysis of the data. The experimental evidence resulted in rejecting the Law of Times as an accurate predictor of the drying time of soil around a buried cable.**

***Index Terms* – Soil Thermal Stability, Soil Thermal Resistivity, Cable Ampacity, Underground Cable Design, Law of Times**

I. Introduction

An underground cable carrying current must dissipate heat to the surrounding soil. The rate at which this heat can be carried away is related to the thermal resistivity of the soil surrounding the cable. The higher the soil's thermal resistivity the more slowly heat passes from the cable to its surroundings and the higher the operating temperature of the cable will be. There is a temperature limit for each cable type above which the cable or its insulation will sustain damage.

In underground cable sizing calculations some value must be used for the thermal resistivity of the surrounding soil. Once this value is known the amount of current the cable can carry before overheating may be determined. The value for thermal resistivity ρ often given in cable ampacity tables such as those in the National Electrical Code is 90 cm °C/W [1]. While some engineers may simply use this assumed value in their cable ampacity calculations, in practice soil ρ may vary considerably making specific measurements necessary especially where important or long underground cables are installed. Another assumption often made is that soil thermal resistivity remains constant. While this assumption may simplify underground cable ampacity calculations it does not hold true in many cases.

The most important factor causing the thermal resistivity of soil to vary is its moisture content. As soil dries out its thermal resistivity increases. The heat generated in a cable can cause moisture to leave the vicinity of the cable, drying out the soil surrounding the cable, thus raising its thermal resistance and impeding the flow of heat away from the cable. This causes the cable temperature to rise and may result in cable damage. The ability of soil to maintain a constant moisture level, and thus a constant thermal resistance, is known as soil thermal stability [2].

II. The Law of Times

Many sources suggest that if the time it takes to dry soil around a heat source of a known diameter is measured, this time can be used to determine the time it will take to dry soil around a heat source of any other diameter [2][3][4][5]. Equation 1 is used by these sources for calculating this time to dry. This equation is known as the "Law of Times" [6].

$$\frac{t_1}{t_2} = \left(\frac{d_1}{d_2}\right)^2 \qquad (1)$$

Equation 1 says that if we know the time, t_1, it takes soil to dry using a heat source of diameter d_1, we can predict t_2, the time it will take the soil to dry around a heat source of diameter d_2, assuming the same heat rate is used for both heat sources.

For example, if we heated a soil sample using a probe with a diameter of 3.1mm, the standard heat probe suggested for thermal resistivity testing [7], and determined the effective drying time was 15 minutes, and wanted to predict the drying time in this soil at this heat rate for a cable of 1 inch (25.4mm) diameter, the expected drying time would be using Equation 1 [7] [8]:

$$\frac{15}{t_2} = \left(\frac{3.1}{25.4}\right)^2$$

$$t_2 = 1{,}007 \quad \text{minutes}$$

So this method would claim that if it took 15 minutes to dry the soil around a 3.1mm source, it would take 1,007 minutes (16 hours) to dry the soil around a cable 1 inch in diameter using the same heat input. The sources also suggest that if the drying time calculated for the cable of interest is long enough, that the soil can be considered thermally stable and the thermal resistivity change due to drying of the soil will not be important in the operation of the cable.

While this method may be simple to use, the question arises as to the theory it is based upon and whether it can yield accurate results. This paper will examine those questions.

III. Theory of the Law of Times

One source [5] states when describing the origin of the Law of Times: "The response of a cable buried in the same soil with the same heat dissipation rate would be related to the probe response through the Fourier number." While this is true, a closer examination of the Fourier number calls into question whether it is applicable when the heat leaving the cable is carried by moisture movement rather than heat conduction through the soil [9].

The Fourier number F_{oh} is defined as [10]:

$$F_{oh} = \frac{\alpha t}{L^2}$$

Where α=thermal diffusivity cm^2/s
L=length of thermal path cm
t=time

Thermal diffusivity is defined as:

$$\alpha = \frac{\text{conductivity}}{(\text{specific heat})(\text{density})}$$

The definition of the Fourier number, and the derivation of the Law of Times given in section 7.15 of Ingersoll [6] both show that the law of times is meant to be used to determine how quickly a particular temperature will be reached at some distance from a heat source if the time needed to reach that temperature at different distance from the heat source was already known. According to the theory this time is dependent upon the thermal conductivity and the specific heat and density of the material being heated. The time it takes for any two points to reach the same temperature is a function of the square of their distances from the heat source.

Nothing in the law considers the effects of heat being carried away from the source by moisture vaporization and migration. The law is concerned only with temperature changes due to heat flow by conduction through the soil or absorption by the soil. For this reason the ability of the Law of Times to predict the amount of drying that will occur in the soil or the resulting change in thermal resistivity is called into question.

IV. Experimental testing of the method

An experiment was designed and performed to determine whether the Law of Times can be used to predict the drying time of soil as a function of heat source diameter. The experiment designed was a single factor, two level procedure with five replicates at each level. The factor tested was the diameter of the heat source and the response variable was the drying time of the soil and the amount of moisture that left the soil sample.

Two thermal heat probes were prepared as described in IEEE and ASTM sources for testing soil thermal resistivity [7][8]. The only difference between the two probes was their diameter. One probe was 3.1mm in diameter and the second was 6.7mm in diameter.

The soil samples used consisted of clean builder's sand. An amount of sand sufficient to split into ten identical samples was collected and mixed with water to achieve a uniform moisture content of 10%. The sand was then divided into ten equal portions and placed into ten identical polyvinyl chloride (PVC) cylinders to make ten samples of sand in as nearly equal volumes and masses as possible. The cylinders were sealed in plastic to prevent moisture loss and were not unsealed until the day they were tested.

The cylinders were numbered from 1 to 10 and each cylinder was assigned a probe size at random using a random number generator. The 3.1mm probe was assigned to five of the samples and the 6.7mm probe was assigned to the other five cylinders.

Each cylinder was then assigned another random number to determine the order the samples with their assigned probe sizes would be tested. The testing consisted of inserting a probe into the sample, turning on an electric current to energize the heater in the probe which injected heat into the soil sample at a known rate. The temperature of the probe was measured during the 14 hour test period. Each sample was tested with the same amount of current under similar testing conditions.

The effective dry time was measured and recorded for each sample using the method suggested by several sources [2] [3]] [4] [5]. Soil thermal resistivity was measured before and after the effective dry time is reached [7]. The mass of water lost by each sample during testing was also measured and recorded. The measurements taken during the testing are shown in Table 1 and Table 2.

Table 1. Measurement from tests

Sample Number	Testing Order	Soil Mass (grams)	Heat Rate (W/cm)	Wet ρ (cm°C/W)	Dry ρ (cm°C/W)
1	6	729.4	0.4195	94.6	184.5
2	5	748.3	0.4213	90.5	170.7
3	9	757.7	0.3995	82.8	161.0
4	7	778.4	0.4229	85.5	155.3
5	10	765.7	0.4174	83.8	154.2
6	4	772.0	0.4169	76.2	170.5
7	3	734.1	0.4140	101.1	167.2
8	1	777.3	0.4159	61.9	120.5
9	8	740.9	0.4164	104.9	173.7
10	2	737.5	0.4152	89.0	162.0
AVERAGE		754.1	0.4160	87.0	162.0

V. Analysis of Data

The drying time for each sample with the applicable test probe size is shown in the first two columns of Table 3. The Law of Times was applied to the measured data from the 6.7mm probe samples in column 2 to predict the drying time for the 3.1mm probe. Column 3 contains the time a 3.1mm probe should take to dry the soil according to the Law of Times given the time to dry measurements for the 6.7mm probe samples in column 2. The measurements for each sample are plotted in Figure 1.

TABLE 2. MEASUREMENTS FROM TESTS.

Sample Number	Probe Size (mm)	Water Lost (grams)	Dry Time (seconds)
1	6.7	10.9	818
2	6.7	11.8	758
3	3.1	14.5	591
4	6.7	14.6	655
5	6.7	12.7	685
6	3.1	14.9	631
7	6.7	12.1	767
8	3.1	12.6	774
9	3.1	11.0	758
10	3.1	12.2	744
AVERAGE		12.7	718

TABLE 3. TIME TO DRY MEASUREMENTS FOR EACH PROBE.

	y_1 3.1mm probe dry time (seconds)	y_2 6.7mm probe dry time (seconds)	y_1 predicted Dry time predicted for 3.1mm probe (seconds)
	591	818	175
	631	758	162
	774	655	140
	758	685	147
	744	767	164
Avg.	699.6	736.6	157.6

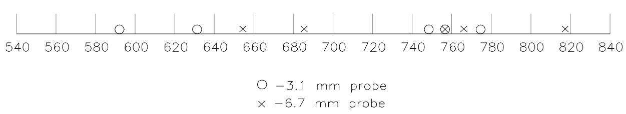

Figure 1. Plot of time to dry for each probe.

A statistical comparison using a two-sample t distribution test was performed on the data in Table 3. If drying times are related as predicted by the Law of Times the mean of the population of all measurements taken with the 3.1mm probe (column 1 of Table 3) would be expected to equal to the mean of the population of all measurements taken with the 6.7mm probe then adjusted using the Law of Times to predict what the drying time should be for the 3.1mm probe (column 3 of Table 3). The first statistical test that will be made is a comparison of the means of the samples in columns 1 and 3 of Table 3. If the Law of Times can be applied as suggested, the means of the data in these two columns should be statistically the same.

The test statistic for the two-sample t-test is Equation 2 [11].

$$t_0 = \frac{\overline{y_1} - \overline{y_2}}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \qquad (2)$$

Where:

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

S_1^2 and S_2^2 are the sample variances found by:

$$S_x^2 = \frac{\sum_{i=1}^{n}\left(y_i - \overline{y}\right)^2}{n - 1} \qquad (3)$$

Where:

$\overline{y_1}$ and $\overline{y_2}$ = the sample means

n = then number of samples in each sample set

y = individual samples

μ = population means

x = the sample set, 1 or 2

The null hypothesis H_0 is that the population means of the 3.1mm values in column 1 of Table 3 will equal the population means of the 6.7mm values adjusted using the Law of Times, column 3 of Table 3. The alternative hypothesis H_1 is that these to population means are not equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where :

μ_1 = The population mean of the 3.1mm drying times

μ_2 = The population mean of the 6.7mm probe drying times adjusted using the Law of Times

If the two means are equal and H_0 is accepted then the Law of Times successfully predicted the drying time. If the two means are not equal and H_1 is accepted, then the Law of Times did not correctly predict the drying rate.

The test statistic from Equation 2 calculated from the measured data will be compared with the reference distribution (the t distribution) $t_{\alpha/2,n1+n2-2}$ where 1-α is the significance level. Using a 95% significance level H_0 will be rejected if $t_0 > t_{0.025,\,8}$ or if $t_0 < -t_{0.025,\,8}$. From a t distribution table it is found that $t_{0.025,\,8} = 2.306$ [11].

Applying Equations 2 and 3 to Columns 1 and 3 of Table 3 the following values can be calculated.

$$S_1^2 = 6854.3$$
$$S_2^2 = 196.3$$
$$S_p = 59.37$$
$$t_0 = 14.43$$

Since 14.43>2.306 there is evidence that the null hypothesis should be rejected at the 95% significance level. Applying the statistical "P" test it can be shown that there is evidence that the null hypothesis should be rejected for all significance levels less than 99.995%. So H_0 must be rejected and the experiment provides evidence that the Law of Times did not properly predict drying time values for the 3.1mm probe.

If the probe diameter has no effect on the drying times the population means for the drying times using the 3.1mm and 6.7mm probes should be identical. To examine this hypothesis the mean values in columns 1 and 2 of Table 3 should be statistically compared and should be identical if the probe diameter had no effect on drying time. The null hypothesis H_0 for this test would be that the population mean of the 3.1mm probe dry times would equal the population mean of the 6.7mm dry times. The alternative hypothesis H_1 would be that the two population means would not be equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where:

μ_1 = The population mean of the 3.1mm drying times

μ_2 = The population mean of the 6.7mm probe drying times

Applying Equations 2 and 3 to the data in columns 1 and 2 of Table 3 the following values are found.

$$S_1^2 = 6854.3$$
$$S_2^2 = 4332.3$$
$$S_p = 74.79$$
$$t_0 = -0.78$$

The t test statistic will once again be $t_{0.025,\,8}$ =2.306 at the 95% significance level.

Since the value of t_0 is within the boundaries of the test statistic, i.e. -2.306<-0.78<2.306 H_0 cannot be rejected at the 95% significance level. By applying the P test it can be found that the null hypothesis would be accepted for all significance levels greater than 54.2%. The experiment appears to support the null hypothesis that the probe diameter has no effect on the effective drying times of the soil since there is no statistical difference between the means of the drying times for the 3.1mm probe and the 6.7mm probe at the 95% significance level.

A similar analysis was done comparing the amount of water that was evaporated during the test for each probe size. This data for each sample is shown in Table 4.

TABLE 4. WATER LOST DURING EACH TEST.

Water Lost 3.1mm probe (g)		Water Lost 6.7mm probe (g)
	14.5	10.9
	14.9	11.8
	12.6	14.6
	11.0	12.7
	12.2	12.1
AVG.	13.0	12.4

The null hypothesis to be tested is that the mean of the mass of water that was lost using the 3.1mm probe equals the mean of the water lost using the 6.7mm probe. The alternative hypothesis is that the two means are not equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where:

μ_1 = The population mean water loss using the 3.1mm probe

μ_2 = The population mean water loss using the 6.7mm probe

Using the same statistical method as before:

$$S_1^2 = 2.66$$
$$S_2^2 = 1.91$$
$$S_p = 1.51$$
$$t_0 = 0.63$$

At the 95% significance level, the test statistic is again $t_{0.025,8}$ =2.306 and the comparison of t_0 calculated from the data with the test statistic results in:

$$-2.306 < 0.63 < 2.306$$

Once again there is no evidence that the null hypothesis should be rejected. Performing the statistical P test shows that for any confidence limit above 45.4% the null hypothesis cannot be rejected. The experiment appears to support the hypothesis that the diameter of the probe has no effect on the amount of drying that occurred in the 14 hour test period for each sample.

VI. CONCLUSIONS

It has been common practice to use the Law of Times to try to determine how quickly soil around a buried cable will dry when the cable is conducting current and generating heat. The recommended method was to obtain a soil sample and test it in the laboratory using a heated probe of a known diameter and measure the effective dry time of the sample. The diameter of the cable being considered, the diameter of the laboratory reference probe, and the effective dry time measured using that probe were used in the Law of Times equation to determine the expected dry time for a heat source, such as a cable, with a different diameter. If this calculated time was long enough it was concluded that the soil thermal stability was adequate for that cable and thermal resistivity changes due to soil drying should not be a factor in calculating the ampacity of the cable

The evidence obtained in the experiment described in this paper discredits this method as an accurate predictor of soil stability. This Law of Times is useful when the temperature of the material under consideration is only affected by the thermal conductivity and specific heat of the material, however, in the case of drying soil significant heat is also being absorbed and carried away by the movement of moisture. This goes beyond the theory upon which the Law of Times is based. The Law of Times would not seem to apply to an underground cable that is drying the surrounding soil.

The evidence obtained in this experiment indicates that the diameter of the heat source does not influence the drying time of the soil or the amount of moisture that is evaporated. Since the Law of Times cannot be used to characterize the thermal stability of the soil, other methods must be used to include soil thermal stability in the ampacity calculations that are done when determining the amount of current an underground cable can safely carry [9].

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VITA

Keith Malmedal received his BSEET degree from Metropolitan State College of Denver in 1995, a MSEE degree (Power), a MSCE degree (Structures) from the University of Colorado- Denver in 1998 and 2002, respectively. In 2008 he received his PhD at Colorado School of Mines, Golden, CO. He has over 24 years combined experience in electrical power system design and system study, teaching, and research, and is presently the President of NEI Electric Power Engineering, Arvada, Colorado, a consulting firm specializing in all aspects of power system engineering and design. He has published over 25 technical papers and taught university courses and short courses related to power systems, machines, protection, renewable energy, and energy policy issues. Dr. Malmedal is also a member of the American Society of Civil Engineers (ASCE) and a registered professional engineer in 25 states (US), and three Canadian provinces.

Carson Bates (M'09) received the B.S. degree in engineering with electrical specialty Magna Cum Laude in 2010, and the M.S. in electrical engineering in 2013 both from the Colorado School of Mines, Golden, CO. He worked as an intern at the National Renewable Energy Laboratory (NREL) in the Advanced Power Electronics for Vehicles group. His work primarily involved investigating heat transfer through IGBT packages. He currently works as an electrical engineer at NEI Electric Power Engineering in the design of substations and industrial facilities and electrical studies of all types.

David Cain received his Associates of Applied Science degree with two Certificates of Completion from the Community College of Aurora. He is a registered electrical apprentice, with three years of field experience and is currently David is attending the University of Colorado at Denver majoring in Electrical Engineering and expects to graduate with a BSEE by 2016.