On the Use of the Law of Times in Calculating Soil Thermal Stability and Underground Cable Ampacity

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Abstract -- **The heat generated by underground cables may cause the soil around the cables to dry, increasing its thermal resistivity and potentially causing the cables to overheat. The ability of soil to maintain a constant resistivity while being subjected to a heat source is known as its "thermal stability". A method using the Law of Times has been recommended by some sources to determine soil stability. To test whether this method can accurately predict soil thermal stability an experiment was performed that tested the hypothesis fundamental to the Law of Times that the diameter of the heat source affects the drying time of the soil surrounding it. This paper reports the results of that experiment and includes the statistical analysis of the data. The experimental evidence resulted in rejecting the Law of Times as an accurate predictor of the drying time of soil surrounding a buried cable.**

Index Terms—**Power Cables, Power Cable Thermal Factors, Thermoresistivity, Underground Power Distribution, Underground Power Transmission, Soil**

I. INTRODUCTION

An underground cable carrying current must dissipate heat to the surrounding soil. The rate at which this heat can be carried away is related to the thermal resistivity of the soil. The higher the thermal resistivity the more slowly heat can pass from the cable to the surroundings and the higher the operating temperature of the cable will be. There is a temperature limit for each cable type above which the cable or its insulation will sustain damage.

When cable sizes are chosen the value for the thermal resistivity of surrounding soil must be determined to calculate how much current any underground cable can carry before it will overheat. This value suggested for soil thermal resistivity, usually signified by ρ (Rho), in some cable ampacity tables, such as those in the National Electrical Code, is 90 cm °C/W [1].

In the field, soil thermal resistivity may vary considerably from the values used in these tables making specific calculations necessary for many locations, especially where important or long underground cables are installed. While the method for calculating the ampacity of underground cable is simplified by assuming that soil thermal resistivity is the same for all soil types and locations, this assumption is not true.

The most important factor controlling thermal resistivity of soil is its moisture content. As soil dries out its thermal resistivity increases. The heat generated in a cable can cause moisture to leave the vicinity of the cable, drying the soil around the cable, raising the soil's thermal resistance and impeding the flow of heat away from the cable. This causes the cable temperature to rise and may result in cable damage. The ability of soil to maintain a constant moisture level, and thus a constant thermal resistance, is known as soil thermal stability [2].

II. THE LAW OF TIMES

Many sources suggest that if the time it takes soil to dry around a heat source of a known diameter is measured, this time can be used to determine the time it will take soil to dry around a heat source of any other diameter [2][3][4][5].

Equation (1) is given by these sources for calculating this time to dry. This equation is known as the "Law of Times" [6].

$$\frac{t_1}{t_2} = \left(\frac{d_1}{d_2}\right)^2 \tag{1}$$

Equation (1) says that if we know the time t_1 it takes soil to dry at any particular heat rate using a heat source of diameter d_1, we can predict the time t_2 that it will take the soil to dry around a heat source of diameter d_2, assuming the same heat rate is used.

For example, if a soil sample was heated using a probe with a diameter of 0.122 inch (3.1mm), the standard heat probe suggested for thermal resistivity testing [7][8], and it was determined that the effective drying time was 15 minutes, and it was desired to predict the drying time in this soil at this same heat rate for a cable of 1 inch (25.4mm) diameter, the expected drying time would be:

$$\frac{15}{t_2} = \left(\frac{3.1}{25.4}\right)^2$$

$$t_2 = 1{,}007 \text{ minutes}$$

So this method would claim that if it took 15 minutes to dry the soil around a 0.122 inch (3.1mm) source, it would take 1007 minutes (16 hours) to dry the soil around a cable 1-inch in diameter with the same heat rate. It is suggested that if the drying time calculated for the cable of interest is long enough, the soil can be considered thermally stable and drying of the soil will not be important in the operation of the cable.

While this method has the advantage of being simple to use, the question arises as to what theory it is based upon and whether it can yield accurate results. This paper will examine those questions.

III. The Theory of the Law of Times

One source [5] states when describing the origin of the Law of Times: "The response of a cable buried in the same soil with the same heat dissipation rate would be related to the probe response through the Fourier number." While this is true in some contexts, a closer examination of the Fourier number calls into question whether this can be applied to heat leaving soil around the cable due to the movement of moisture out of the area and the inflow of moisture from the surrounding area [9]. The Fourier number F_{oh} is defined as [10]:

$$F_{oh} = \frac{\alpha t}{L^2} \quad (2)$$

Where:
α=thermal diffusivity cm^2/s
L=length of thermal path in cm
t=time in seconds

Thermal diffusivity is defined as:

$$\alpha = \frac{k}{c\rho} = \frac{\text{conductivity}}{(\text{specific heat})(\text{density})} \quad (3)$$

The definition of the Fourier number, and the derivation of the Law of Times given in section 7.15 of Ingersoll [6] both show that the law of times is used to determine how quickly a particular temperature will be reached at some distance from a heat source if the time needed to reach that temperature at a different distance from the heat source is already known. However, according to (2) and (3) and the derivation given in [6] the time calculated using the Law of Times is dependent upon the thermal conductivity, specific heat and density of the material being heated. If these parameters are known then the Law of Times allows the calculation of the time required for any two points to reach the same temperature and states that these times are proportional to the square of their distances from the heat source and the material surrounding that source.

However, neither the Fourier number nor the Law of Times consider the effects of heat flow due to moisture migration. Furthermore, it does not address the time it takes for heat to vaporize moisture and for the moisture to leave the area of the heat source. The Law is concerned with temperature changes only due to heat flow by thermal conductivity and heat absorption due to the material's specific heat. It does not consider the parameter of heat loss by moisture migration. The question is therefore raised: Since the Law of Times does not consider heat flow due to moisture migration, is it appropriate to use the Law of Times to try to predict soil stability as is suggested by some sources?

IV. Experimental Testing of the Method

An experiment was performed to determine whether the Law of Times can be used to predict the drying time of soil. The experiment designed was a single factor, two level procedure with five replicates at each level. The factor tested was the diameter of the heat source and the production variable was the effective drying time of a soil sample.

Two thermal heat probes were prepared along the lines of the instructions given in IEEE and ASTM sources for testing soil thermal resistivity [7][8]. The only difference between the two probes was their diameter. One probe was 0.122 inch (3.1mm) in diameter and the second was 0.264 inch (6.7mm) in diameter.

The soil type used for the experiment was clean sand. A sample of sand was collected and mixed to achieve as homogeneous a mixture as possible. The sand was then divided into ten equal portions and placed into 10 identical 3-inch diameter by 7-inch polyvinyl chloride (PVC) cylinders to make 10 samples of sand in as nearly equal volumes and masses as possible. The cylinders were sealed in plastic to prevent moisture loss and were not unsealed until the day they were tested.

The cylinders were numbered from 1 to 10 and each cylinder was assigned a probe size at random. The 0.122 inch (3.1mm) probe was assigned to five of the samples and the 0.264 inch (6.7mm) probe was assigned to the other five cylinders. These assignments were made using a random number generator.

Each cylinder was then assigned another random number which was used to determine the order in which the samples and their assigned probe sizes would be tested. The sample to be tested on a particular day was chosen, its probe was inserted into the sample, and heat was injected by the probe into the soil sample for a period of 14 hours. The temperature of the probe was measured during the testing period. The heat injected by each probe was held as constant as possible for each sample during the testing period.

The effective dry time was measured and recorded for each sample using the method suggested by several sources [2][3][4][5]. The measurements taken were also used to determine the soil resistivity before and after the effective dry time [7]. A typical graph of recorded data is shown in Fig. 1.

The solid line is the temperature recorded over the 14 hour period. There are two slopes of interest shown in this Figure. The first starts at approximately 100 seconds and continues to approximately 1000 seconds. At that point the slope of the graphs changes and stays constant until approximately 3000 seconds. At that point a third slope begins and lasts until the end of the test.

The first slope between 100 and 1000 seconds can be used to determine the soil's beginning (wet) resistivity [7]. After the soil dries the resistivity changes. This is represented by the change to the second slope between 1000 and 3000 seconds. From this second slope the effective dry soil resistivity can be determined. The transition point between the two slopes is the “effective dry time” of the soil sample at which point the soil changes from wet to dry resistivity. The final slope will slowly approach a steady state condition of constant temperature as the two dimensional and end effects of the heat source become important and the entire test apparatus reaches equilibrium in a two dimensional heat flow configuration. This eventually will include the convection into the air around the test apparatus.

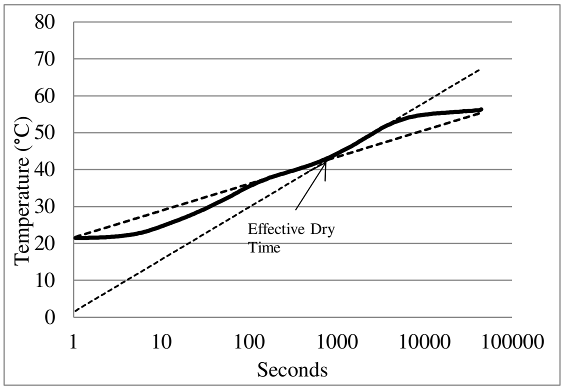

Fig. 1. Graph of recorded data.

The response variable measured for this experiment is the effective time to dry for each sample. The measurements taken during the testing are shown in Table I and Table II.

TABLE I
MEASUREMENT FROM TESTS

Sample Number	Testing Order	Soil Mass (g)	Heat Rate (W/cm)	Wet ρ cm°C/W	Dry ρ cm°C/W
1	6	729.4	0.4195	94.6	184.5
2	5	748.3	0.4213	90.5	170.7
3	9	757.7	0.3995	82.8	161.0
4	7	778.4	0.4229	85.5	155.3
5	10	765.7	0.4174	83.8	154.2
6	4	772.0	0.4169	76.2	170.5
7	3	734.1	0.4140	101.1	167.2
8	1	777.3	0.4159	61.9	120.5
9	8	740.9	0.4164	104.9	173.7
10	2	737.5	0.4152	89.0	162.0
AVERAGE		754.1	0.4160	87.0	162.0

TABLE II
MEASUREMENTS FROM TESTS

Sample Number	Probe Size (mm)	Water Lost (g)	Dry Time (sec)
1	6.7	10.9	818
2	6.7	11.8	758
3	3.1	14.5	591
4	6.7	14.6	655
5	6.7	12.7	685
6	3.1	14.9	631
7	6.7	12.1	767
8	3.1	12.6	774
9	3.1	11.0	758
10	3.1	12.2	744
AVERAGE		12.7	718

V. ANALYSIS OF EXPERIMENTAL DATA

The drying time for the samples tested with each probe is shown in the first two columns of Table III. The third column is the 0.264 inch (6.7mm) time measurements transformed using the Law of Times to the dry time predicted for a 0.122 inch (3.1mm) probe. In other words, column 3 is the time a 0.122 inch (3.1mm) probe should take to dry the soil according to (1) given the time to dry measurement for the 0.254 inch (6.7mm) probe in column 2.

TABLE III
TIME TO DRY MEASUREMENT FOR EACH PROBE

0.122 inch (3.1mm) probe dry time (s)	0.264 inch (6.7mm) probe dry time (s)	0.264 inch (6.7mm) dry time corrected to 0.122 inch (3.1mm) (s)
591	818	175
631	758	162
774	655	140
758	685	147
744	767	164
Avg. 699.6	736.6	157.6

The measurements for each probe are plotted in Fig. 2 and a box plot of the data is shown in Fig. 3.

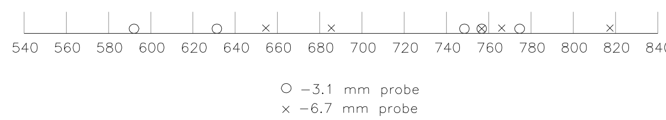

Fig. 2. Plot of time to dry for each probe.

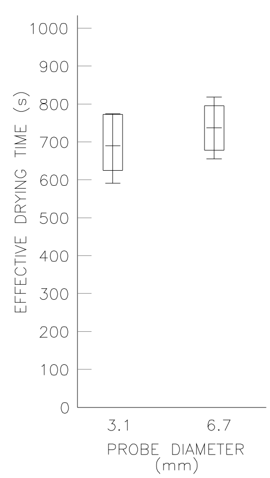

Fig. 3. Box plot of data for each probe.

A normal distribution plot was done for the data taken using each probe. This plot along with the best fit lines for the sampled points is shown in Fig. 4. Fig. 5 shows the normal distribution plot of the residuals from Table III. It appears from these plots that the measurements and the residuals are reasonably normally distributed with similar variances.

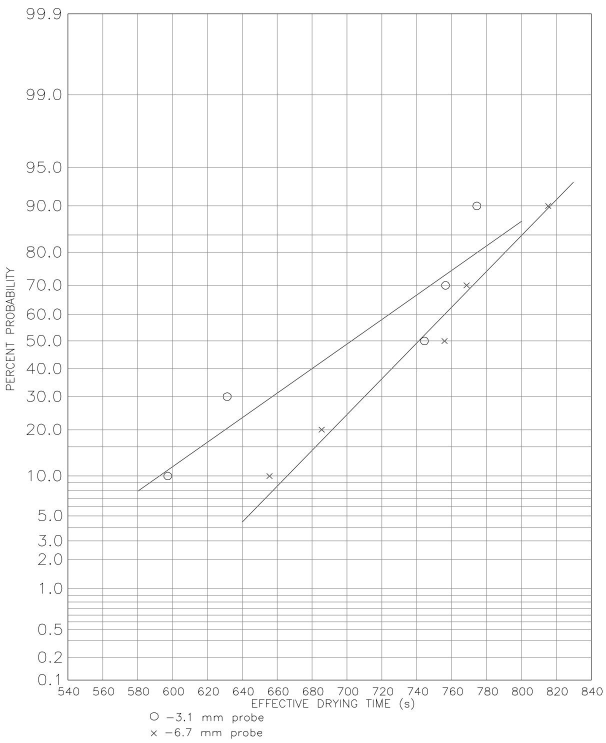

Fig. 4. Normal distribution plot of the data.

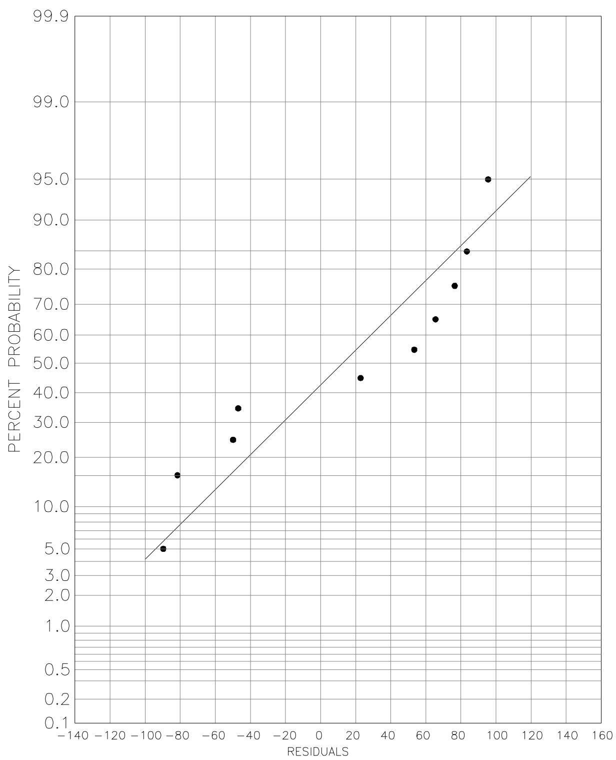

Fig. 5. Normal distribution plot of residuals from Table III.

The drying time measurements were plotted against a variety of abscissas to try to detect any trends in the data due to the experimental method used. A significant trend in the data was discovered and is shown in the plot in Fig. 6. In this Figure the time to dry values were plotted against the test number. It may be seen that there is a definite downward trend of drying times with the later tests.

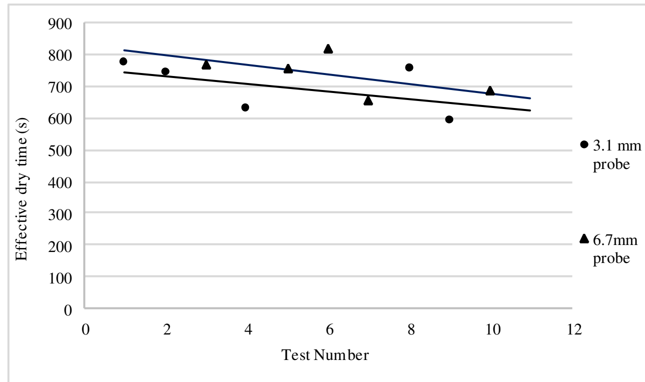

Fig. 6. Dry times plotted against test number.

The most likely explanation for this trend is that the tests were done over a period of two weeks. The later samples had time to lose some moisture that was not due to the heating from the probes. This decreased the time to dry for the samples tested later since some of their moisture had already evaporated. Even though an effort was made to seal the samples against water loss until they were tested, it is clear from this trend that some water was lost as the samples were stored while waiting for testing.

The trend in the data does not appear to be significant enough to affect the final results. The next step is a statistical analysis of the data to determine whether the Law of Times correctly predicted the change in drying times as the probe diameter changed.

A means comparison t test was performed on the data in Table III. The basic question this experiment is designed to test is whether the drying times are related by the Law of Times. If this were true the mean of the population of all measurements taken with the 0.122 inch (3.1mm) probe would be expected to be equal to the mean of the population of all measurements taken with the 0.264 inch (6.7mm) probe as corrected using the Law of Times. So the test data that will be compared first will be the values shown in columns 1 and 3 of Table III. If the Law of Times worked as suggested the mean values of the data in these two columns should be statistically the same.

A two-sample t test was performed on the data in columns 1 and 3 of Table III. The test statistic for this test is [11]:

$$t_0 = \frac{\overline{y_1} - \overline{y_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad (4)$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \qquad (5)$$

S_1^2 and S_2^2 are the sample variances found by:

$$S_x^2 = \frac{\sum_{i-1}^{n}\left(y_i - \overline{y_x}\right)^2}{n-1} \qquad (6)$$

Where:

$\overline{y_1}$ and $\overline{y_2}$ = the sample mean drying time from probe 1 or 2
n = then number of samples in each sample set
y_i = individual sample drying time from probe 1 or 2

The null hypothesis H_0 is that the population means of the 0.122 inch (3.1mm) values in column 1 of Table III will equal the population means of the 0.264 inch (6.7mm) values adjusted using the Law of Times, in column 3 of Table III. The alternative hypothesis H_1 is that these two population means are not equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where :

μ_1 = The population mean of the 0.122 inch (3.1mm) drying t imes
μ_2 = The population mean of the 0.264 inch (6.7mm) probe drying times adjusted using the Law of Times

The test statistic from Equation 2 will be compared with the reference distribution (the t distribution) $t_{\alpha/2,n1+n2-2}$ where α is the significance level. Using a 95% significance level H_0 will be rejected if $t_0 > t_{0.025,8}$ or if $t_0 < -t_{0.025,8}$. From a t distribution table it is found that $t_{0.025,8} = 2.306$.

From (4), (5), and (6), and using the values in Table III:

$$S_1^2 = 6854.3$$
$$S_2^2 = 196.3$$
$$S_p = 59.37$$
$$t_0 = 14.43$$

Since 14.43>2.306 there is evidence that the null hypothesis should be rejected at the 95% significance level. Applying the P test it can be shown that there is evidence that the null hypothesis should be rejected for all significance levels less than 99.995%. So the experiment provides evidence that the mean values of the two data sets are not equal and the Law of Times did not properly predict drying time values that corresponded to the 0.122 inch (3.1mm) measurements found experimentally.

If the probe had no effect on the drying times the population means for the 0.122 inch (3.1mm) and 0.264 inch (6.7mm) probes should be identical. If this hypothesis is statistically tested, the mean values in columns 1 and 2 of Table III should be statistically identical. The null hypothesis for this test is now that the population mean of the 0.122 inch (3.1mm) probe drying times equal the population mean of the 0.264 inch (6.7mm) unaltered drying times, and the alternative hypothesis is that the two population means are not equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where:

μ_1 = The population mean of the 3.1mm drying times
μ_2 = The population mean of the 6.7mm probe drying times

Using (4), (5), and (6) once again and applying the test to the new hypothesis, the following may be found.

$$S_1^2 = 6854.3$$
$$S_2^2 = 4332.3$$
$$S_p = 74.79$$
$$t_0 = -0.78$$

The t test statistic for this test will once again be $t_{0.025,8} = 2.306$ at the 95% significance level. The value of t_0 is within the boundaries of the test statistic, i.e.

$$-2.306 < -0.78 < 2.306$$

The null hypothesis is therefore not rejected at the 95% significance level. By applying the P test it can be found that the null hypothesis would be accepted for all significance levels greater than 54.2%. The experiment appears to support the null hypothesis in this case and the conclusion that the probe diameter has no effect on the effective drying times of the soil. At a significance level of 95% there is no reason to believe there is difference between the means of the drying times for the 0.122 inch (3.1mm) probe and the 0.264 inch (6.7mm) probe.

A similar analysis was done on the amount of water that was evaporated during the test for each probe size. This data is shown in Table IV.

TABLE IV
WATER LOST DURING EACH TEST.

	Water Lost 0.122 inch (3.1mm) probe (g)	Water Lost 0.264 inch (6.7mm) probe (g)
	14.5	10.9
	14.9	11.8
	12.6	14.6
	11.0	12.7
	12.2	12.1
AVG.	13.0	12.4

The null hypothesis that will now be tested is that the mean of the water mass that was lost when the 0.122 inch (3.1mm) probe was used is equivalent to the mean of the water mass that was lost when the 0.264 inch (6.7mm) probe was used. The alternative hypothesis is that the two means are not equal.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

Where :

μ_1 = The mean water loss using the 0.122 inch (3.1mm) probe
μ_2 = The mean water loss using the 0.264 inch (6.7mm) probe

Using (4), (5), and (6), and the measurements in Table IV:

$$S_1^2 = 2.66$$
$$S_2^2 = 1.91$$
$$S_p = 1.51$$
$$t_0 = 0.63$$

Using the 95% significance level, the test statistic is again $t_{0.025,8}$ =2.306 and the comparison of t_0 from the data with the t test statistic results is:

$$-2.306 < 0.63 < 2.306$$

So once again there is no evidence that the null hypothesis should be rejected and the P test shows that this is true for any confidence limit above 45.4%. The experiment appears to support the hypothesis that the diameter of the probe has no effect on the amount of drying that occurred in the 14 hour test period.

VI. CONCLUSIONS

The Law of Times has been used in the past to try to determine how quickly soil around a buried cable will dry when the cable is conducting current and generating heat. In the sources recommending its use, it is assumed that the law holds true for all soil types. The method involved obtaining a soil sample and testing it in the laboratory using a heated probe of the same size and type as the 0.122 inch (3.1 mm) probe used in the experiment reported herein. The time for the soil sample to dry using this probe was measured. The diameter of the cable being considered, the diameter of the probe used in testing, and the time to dry that was measured in the laboratory for the reference probe, are all plugged into the Law of Times equation to determine the expected time it will take heat generated by the cable to dry out the soil. In this way an attempt was made to determine the soil thermal stability and its resulting effect on the temperature of the cable.

The empirical evidence obtained in this experiment discredits this method as an accurate predictor of soil stability. This Law of Times is useful when the temperature of the material under consideration is only affected by the conductivity of the material and the material's specific heat and mass. However, in most underground cable installations significant heat is being absorbed and carried away by the movement of moisture through the soil in addition to heat loss by conduction and the absorption of heat. So the Law of Times would not apply to the case of heat generated in an underground cable which dries the surrounding soil.

The experimental evidence shows that the diameter of the heat source does not influence drying time of the soil or the amount of moisture that is evaporated, in the way predicted by the Law of Times. It is questionable from both experimental and theoretical perspectives that the Law of Times should be applied to predict soil drying times and soil thermal stability. Other methods must be used to include soil thermal stability in cable ampacity calculations [9].

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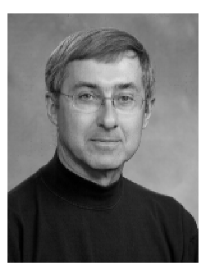
VITA



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